Numerical analysis of a mixed convection in a vertical channel filled with a heat generating porous medium with local thermal non-equilibrium

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Abstract—The purpose of this work is to study numerically the mixed heat convection inside a vertical channel filled with a porous medium which produces internal heat generation and in the case of local thermal non-equilibrium conditions. Therefore two heat transport equations are adopted, one for each phase. The numerical solution is obtained by using the finite volume method and results are established concerning the effects of heat generation rate, Biot interstitial number, Reynolds number, and Raleigh number. The results particularly show that for high internal heat generation in the solid, the local thermal equilibrium model can be invalid. Moreover, it was highlighted that the local transfer coefficient value may also be essential to confirm or invalidate the local equilibrium hypothesis.

Keywords—Heat generation, local thermal non-equilibrium, mixed convection, porous medium.

I. INTRODUCTION

It is well known that the quest to understand the convective heat transfer in porous media is constantly growing. This can be attributed to the great interest of many researchers to estimate the heat and fluid flow rates and behaviour through porous medium enclosed in various geometries. This large interest is easily understandable since porous media are used nowadays in vast applications which cover many engineering disciplines. More applications and good understanding of the subject are given in the work of Nield and Bejan [1]. It is usually assumed that the temperature fields of the solid and fluid phases are identical locally; such a situation is generally known as local thermal equilibrium (LTE). The opposite situation is known as local thermal non-equilibrium (LTNE), and in the last case the solid matrix may have a different temperature from that of the saturating fluid. Therefore two heat transport equations must be adopted, one for each phase. Plenty of works have been carried out considering the LTE model, whereas the LTNE model has not received as much attention as LTE model.

Among the studies closest to the considered case one can quote the following works. Thus, A. Abedou and K. Bouhadeef [2] analyzed heat transfer by forced convection in a porous channel using thermal non equilibrium model. They have developed a mapping to delimitate areas of validity of local thermal equilibrium, depending on the conductivity ratio and on interstitial Biot number. D. E. Ameziani et al. [3] conducted a comprehensive study of natural convection from a vertical open ended porous duct. They highlighted the delicate treatment of the output dynamic condition at the end of the cylinder. Free convection over a vertical permeable flat plate in a porous medium with internal heat generation has been studied by A. Postelnicu et al.[4]. Arun Narasimhan, and B.V.K. Reddy [5] have investigated the forced convection in a heat generating bi-disperse porous medium channel. Irfan Anjum et al. [6] have considered the non-equilibrium model to analyse the heat transfer through vertical annulus embedded with porous medium. A. Bousri et al. [7] have performed a numerical study of heat and mass transfer for a constant heat source in a porous medium has been presented by O.M. Haddad et al. [9] while Nawaf H. Saeid [10] presented a numerical analysis of mixed convection in a vertical porous layer using non-equilibrium model. Finally, Kok-Cheong Wong and Nawaf H. Saeid [11] discussed the effect of local thermal non-equilibrium on the mixed convection on jet impingement cooling in a horizontal porous layer.
The present work is aimed to study the mixed convection in a vertical channel filled with heat generating porous medium, using a two-temperature thermal non-equilibrium model. The principal goal is to search for ranges of governing parameters which allow adopt or not the assumption of local thermal equilibrium.

II. MATHEMATICAL FORMULATION

The problem under investigation is the convective heat transfer in a vertical channel bounded by two parallel plates and filled with a porous medium with internal heat generation. The schematic of this system is given in Fig. 1, where H and L denote the height and length of the channel respectively. The inlet temperature and velocity are \( T_0 \) and \( V_0 \) respectively; whereas the plates are kept at a constant temperature \( T_p \) (\( T_p > T_0 \)). The following assumptions are applied:

(a) The flow in the channel is 2 D, steady, laminar and incompressible.
(b) The porous medium is homogeneous and isotropic and saturated with the flowing fluid.
(c) The properties of the two phases are constant.

Under these assumptions, the conservation equations for mass, momentum and energy for the two-dimensional thermal non-equilibrium model are given by:

Continuity equation:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

Momentum equations:
\[
\rho_f \left( \frac{u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_{eff} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\mu}{K} u
\]  

Energy equations:

- Solid phase:
\[
0 = (1 - \varepsilon) k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) T_s + (1 - \varepsilon) Q - h_{sf} a_{sf} (T_s - T_f)
\]  

- Fluid phase:
\[
\varepsilon (\rho c_p) \left( \frac{u}{\partial x} + v \frac{\partial T_f}{\partial y} \right) = \varepsilon k_f \left( \frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right) T_f + h_{sf} a_{sf} (T_s - T_f)
\]  

Where \( u, v \) are the velocity components along x and y axis, \( T_f \) is the fluid phase temperature, \( T_s \) is the solid phase temperature, \( a_{sf} \) is the interstitial surface area per unit volume of the porous medium, \( \beta \) is the fluid volumetric expansion coefficient, \( h_{sf} \) is the interstitial heat transfer coefficient, \( \mu_{eff} \) is the effective viscosity, which will be assimilated to the dynamic viscosity \( \mu_f \) of the fluid (Brinkman assumption) and \( \rho_f \) is the fluid density.

Here non-dimensional variables are introduced:

\[
X = \frac{x}{L}; \quad Y = \frac{y}{L}; \quad U = \frac{u}{V_0}; \quad V = \frac{v}{V_0}; \quad \theta = \frac{T - T_0}{T_p - T_0};
\]

\[
H = \frac{h}{L}; \quad P = \frac{\varepsilon \rho}{\rho_f V_0^2}
\]

Substituting (6) into (1)-(5), we obtain the following dimensionless governing equations:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

\[
\left( U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) U - \frac{1}{Re Da} U
\]
\[
\left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y}\right) = -\frac{\partial P}{\partial Y} + \frac{1}{\text{Re}} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) V - \frac{1}{\text{Re} \cdot \text{Da}} V + \frac{\text{Ra} \cdot \varepsilon}{\text{Re}^2 \cdot \text{Pr}^f} \theta_f
\]

\[
U \frac{\partial \theta_f}{\partial X} + V \frac{\partial \theta_f}{\partial Y} = \frac{1}{\text{Re} \cdot \text{Pr}^f} \left(\frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2}\right) \theta_f + \frac{6(1-\varepsilon)}{\varepsilon^2} \frac{\text{Bi}}{\text{Re}_p \cdot \text{Pr}^f} \left(\theta_s - \theta_f\right)
\]

\[
0 = \left(\frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2}\right) \theta_f + \text{Re} \cdot q^w = \frac{6 \text{Bi} \cdot \text{Re}^f}{\varepsilon} \left(\frac{\text{Re}}{\text{Re}_p}\right) \left(\theta_s - \theta_f\right)
\]

Where, the parameters arising in the non dimensionalization are defined as:

- Reynolds number: \(\text{Re} = \frac{\rho_f L V_0}{\mu_f}\)
- Darcy number: \(\text{Da} = \frac{K}{\varepsilon L^2}\)
- Prandtl number: \(\text{Pr} = \frac{\nu_f}{\alpha_f}\)
- Biot number: \(\text{Bi} = \frac{h_{sf} L}{k_f}\)
- Rayleigh number: \(\text{Ra} = \frac{g \beta \Delta T_{ref} L^3}{\nu_f \alpha_f}\)
- Volumetric heat generation: \(q^w = \frac{Q}{Q_{ref}}\)

Where: \(Q_{ref} = \frac{k_f \Delta T_{ref}}{L^2}\), and \(\Delta T_{ref} = T_p - T_b\)

Afterward, the boundary conditions in dimensionless form are:

At the inlet (\(Y = 0\))

\[
U(X,0) = 0; \quad V(X,0) = V_0; \quad \theta_s(X,0) = \theta_f(X,0) = 0
\]

At the exit (\(Y = H\))

\[
\frac{\partial U(X,H)}{\partial Y} = \frac{\partial V(X,H)}{\partial Y} = 0; \quad \frac{\partial \theta_s(X,H)}{\partial Y} = \frac{\partial \theta_f(X,H)}{\partial Y} = 0
\]

In the middle (\(X = 0\))

\[
\frac{\partial U(0,Y)}{\partial X} = \frac{\partial V(0,Y)}{\partial X} = 0; \quad \frac{\partial \theta_s(0,Y)}{\partial X} = \frac{\partial \theta_f(0,Y)}{\partial X} = 0
\]

On the wall (\(X = 0.5\))

\[
U(0.5,Y) = V(0.5,Y) = 0, \quad \theta_s(0.5,Y) = \theta_f(0.5,Y) = 1
\]

Thereafter, the average temperatures for both solid and fluid phases are defined as:

\[
\theta_{fm} = \frac{1}{\frac{1}{2} V_m \int_0^1 V \theta_f \, dx}; \quad \theta_{sm} = \frac{1}{\frac{1}{2} V_m \int_0^1 V \theta_s \, dx}
\]

Where \(V_m\) is the mean velocity:

\[
V_m = \frac{1}{\frac{1}{2} \int_0^1 V \, dX}
\]

### III. Numerical Procedure

The momentum equations (8)–(9) and energy equations (10)–(11) are integrated numerically over control volumes using the finite volume method [12]. The power law scheme [13] is applied for the convection–diffusion formulation. The SIMPLER algorithm [12] for the pressure velocity coupling is used in the present study.

As the flow and heat transfer characteristics are symmetrical around \(Y\) axis as shown in Fig.1. Hence, one half of the domain is considered to be the computational area. Taking into account the structure of the domain, and after tests on the sensitivity to the grid, we chose a number of 121x241 nodes, uniform in the both directions.

### IV. Results and Discussion

In all of these results, the Prandtl number \(\text{Pr}\) is considered at the value of air (0.7) and the porosity of the porous medium is taken equal to 0.9 (foamy solid phase, for example).

The effect of the Biot number on the temperature profiles is illustrated on Fig. 2. It appears that for small values of Biot, the temperature difference between the solid and fluid phases is very important due to the low heat transfer between the solid and fluid phases. On the other hand, when the Biot number becomes significant (\(\text{Bi}=10\)), the temperature difference
between fluid and solid phases decreases, until the two profiles overlap, consequence of the better thermal communication between fluid and solid phases.

![Temperature Profile](image1)

**Fig. 2** Non dimensional temperature profile versus axial direction for \( R_c=1, q''=1, Re=10, Da=10^{-4} \) and \( Ra=10^4 \).

The variation of average temperature for both solid and fluid along the channel for different values of Biot number is presented in Fig. 3; we note that for low values of Biot number, with low heat generation Fig. 3 (a), the average solid temperature reached values close to the temperature of the wall. Increasing heat generation Fig. 3 (b) and (c), tends to enhance the average temperature of the solid phase which can take very high values. This causes an increase in the difference between the temperatures of the two phases and can be explained by the poor heat transfer between the solid and the fluid.

As the value of Biot number increases, the difference between the temperatures of both solid and fluid phases becomes smaller, what causes approaching towards the thermal equilibrium case. This is due to the fact that increasing the heat transfer coefficient parameter between the solid and the fluid phases leads to better heat transfer between them, bringing the temperature of the fluid closer to that of solid.

![Average Temperature](image2)

**Fig. 3** Variation of average temperature along the channel for \( Re=1, Re=10, Da=10^{-4} \) and \( Ra=10^4 \).

(a) \( q''=1 \), (b) \( q''=10 \), (c) \( q''=100 \)

Fig. 4 shows the variation of average Nusselt numbers of solid and fluid phases versus Darcy number, for different values of the Rayleigh number.

![Average Nusselt Numbers](image3)

For low value of Reynolds number, Fig. 4 (a), we note that when the Rayleigh number is weak, the Darcy number variation has no significant influence on the Nusselt numbers of the two phases. Otherwise, when the Rayleigh number increases the average Nusselt numbers of both phases decrease with increasing Darcy number, and the transfer is higher because of low permeability. In this case, the thicknesses of boundary layers are smaller and the thermal gradients become more important, which increases the transfer coefficients.
However increasing Biot number tends to decrease the difference between the two temperatures, which becomes negligible.

For situations of high internal energy generation in the solid, the average temperature of the solid phase will reach high values, and the difference between the temperatures of the two phases increases.

Finally, it is important to conclude that the application of local thermal equilibrium model may be non correct in some situations, especially when the values of some governing parameters can make the temperature difference between the two phases be important.

**References**


