Abstract—In this paper, we will present a new and a simple method based on an Unknown Input Observer (UIO) to detect in which configuration the hybrid dynamic system (HDS) is, at a given instant, where the evolution of the discrete part is governed by a Petri net. Assuming that some switching conditions are known and the other switches are considered as unknown inputs for the HDS. Therefore, an UIO is designed in order to estimate both the marking places and the firing vector of transitions and it guarantees exponential convergence of the state estimation error to zero. The sufficient conditions for the existence of the observer are also presented. Finally, simulation results demonstrate the efficiency of the proposed observer.

Keywords— Hybrid Dynamic Systems; Petri Nets; Unknown Input observer.

I. INTRODUCTION

The state estimation problem for hybrid dynamical systems remains a challenging problem, particularly when the discrete mode needs to be identified and an estimation of the continuous state vector should be also given in the same time. In this context, various studies have focused on the design of observers for hybrid systems [5], [6] and [11]. In [1] the authors considered the problem of reconstructing the continuous state vector for hybrid systems, assuming that the discrete state is known at each time. A more difficult case, when the discrete mode is not known was presented in [4] for the design of dynamical observers of hybrid systems that reconstructs the discrete state and the continuous one from the knowledge of the continuous and the discrete outputs. Also, in [12] hybrid observers are designed for linear switched systems modeled via Differential Petri Nets, the convergence of the hybrid observer was proved through a Linear Matrix Inequality (LMI) conditions.

In the framework of the discrete state estimation, the problem of estimating the discrete state of a class of hybrid systems is addressed in [10]. An approach is presented in [14] to construct the discrete state for a class of nonlinear switched systems affected by model uncertainties by assuming that the continuous state is available for measurement. A method to detect the active mode for hybrid mechatronics systems is presented in [2] and for hybrid photovoltaic systems in [3].

In this paper, we focus on the detection of the active mode for a class of the hybrid system at a given instant where the evolution of the discrete part is governed by a Petri net and the continuous part described by difference or differential equations. The PN represents the different configurations of the system and with each configuration, a continuous subsystem is associated. The hybrid system considered is characterized by switching laws that can depend on the continuous states and/or external events. A new formal presentation for this class hybrid system is given in order to shown that the hybrid system can have several subsystem evolved at the same time. We will present a new and a simple method based on an UIO for the discrete part of the hybrid system in order to detect in which configuration the system is at a given instant by assuming that some switching conditions are known (for example, the condition of switching dependent on external inputs known, output or measured states of the system). From these switching conditions, some modes are detected. The remaining switching conditions present the unknown inputs for the global system. The modes detected are considered as observables modes by the discrete observer in order to give the complete discrete states estimation.

II. HYBRID SYSTEMS PRESENTATION

As mentioned in the introduction, we are interesting to the hybrid modeling formalism that combines a continuous formalism that describe the evolution of the continuous state under differential or difference equation) with a discrete formalism presented by a PN [8]. The class of HDS considered in this work is described as follow:

\[
\dot{X} = QF(X)
\]  

With \(X\), \(F(X)\) and \(Q\) are respectively, the continuous state vector, the continuous vector dynamics and the location matrix defined as:

\[
X = \begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_n
\end{bmatrix}, \quad F(X) = \begin{bmatrix}
f_1(X_1) \\
f_2(X_2) \\
\vdots \\
f_n(X_n)
\end{bmatrix}
\]
\[ Q = \begin{bmatrix} p_1 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & p_n \end{bmatrix} \]

\[ p_1, p_2, \ldots, p_n \] indicate the different configuration of the system and present also the places of the PN. With this formal presentation, the system can have several subsystems evolved at the same time. The dimensions of the subsystems can be different and defined as follow:

\[ \dim(X_i) = l_i \quad \text{for} \quad i = 1, \ldots, n \]

\[ \dim(X) = \sum_{i=1}^{n} X_i \]

The system (1) is rewritten as the following formula:

\[
\begin{align*}
\dot{X}_1 &= p_1 f_1(X_1) \\
\dot{X}_2 &= p_2 f_2(X_2) \quad \text{with} \quad p_i \in \{0,1\} \quad \text{for} \quad i = 1, \ldots, n \\
& \vdots \\
\dot{X}_n &= p_n f_n(X_n)
\end{align*}
\]

With \( X_i, f_i(X_i) \) and \( p_i \) are defined as:

\[
X_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \vdots \\ x_{in} \end{bmatrix}, \quad f_i(X_i) = \begin{bmatrix} f_{i1}(X_i) \\ f_{i2}(X_i) \\ \vdots \\ f_{in}(X_i) \end{bmatrix}
\]

\[
p_i = \text{diag}(p_{i1}, p_{i2}, \ldots, p_{in}) \quad \text{for} \quad i = 1, \ldots, n
\]

As illustrated in Fig. 1, the PN represents the different configurations of the system. With each configuration, a continuous subsystem is associated in order to represent the significant laws of physics in this specific context. The activation of the subsystems is specified according to the marking of places i.e. when a token is put into a place, the subsystem associated with this place, is activated. The change from one configuration to another is done through the firing of transitions. These transitions are triggered by continuous variables reaching a certain threshold or by external events. In the instant of deactivation, the deactivated system remains constant with its final value or returns to its initial condition.

\[ \dot{X}_1 = f_1(X_1) \]

\[ \dot{X}_2 = f_2(X_2) \]

\[ \dot{X}_3 = f_3(X_3) \]

**Fig. 1. The hybrid system**

The problem addressed in this paper, is to detect the active mode for a class of HDS by determine the location matrix of the system under the assumption that some switching conditions are known (for example, the condition of switching dependent on external inputs known, output or measured states of the system). The remaining switching conditions present the unknown inputs for the global system. These unknown switching are activated at unknown instants and it is therefore impossible to locate the active mode of the system. The main idea is to design an UIO for the discrete part of HDS governed by a PN where the evolution of the token in a PN is given by a state equation similar to that used for discrete dynamic systems.

A PN is a graphical and mathematical modeling tool applicable to many systems and formally defined by a 5-tuple \( PN = (P,T,\text{Pre},\text{Post},M_0) \) where \( P \) is a finite non empty set of \( n \) places; \( T \) is a finite non empty set of \( m \) transitions (with \( PT = \emptyset \)), \( \text{Pre} : P \times T \rightarrow \{0,1\} \) is the input incidence mapping; \( \text{Post} : P \times T \rightarrow \{0,1\} \) is the output incidence mapping and \( M_0 \) is the initial marking. The marking vector of the PN \( SM \) is governed by the following equation [7], [15]:

\[ M_{k+1} = M_k + W \sigma_{k+1} \quad \text{(3)} \]

Where \( M_k \in \mathbb{N}^n \) is the marking vector of places, \( \sigma_k \in \mathbb{N}^m \) is the firing vector at time instant \( k \) (i.e., a vector whose \( i^{th} \) entry denotes the number of times transition \( t_i \) has fired) and \( W \) is the incidence matrix of the PN, it is defined as \( W = \text{Post-Pre} \).

The marking vector and vector transition are arranged as follows:

\[ M_k = \begin{bmatrix} M_k^1 \\ M_k^2 \end{bmatrix} \quad \text{and} \quad \sigma_k = \begin{bmatrix} \sigma_k^1 \\ \sigma_k^2 \end{bmatrix} \quad \text{(4)} \]

Where \( M_k^1 \in \mathbb{N}^{n_1}, M_k^2 \in \mathbb{N}^{n_2} \) respectively, the marking of measured places and unmeasured places, \( \sigma_k^1 \in \mathbb{N}^{m_1}, \sigma_k^2 \in \mathbb{N}^{m_2} \) are respectively the firing count of the measured and unmeasured transitions at time \( k \), with \( n = n_1 + n_2 \), \( m = m_1 + m_2 \).

Now, we decompose the incidence matrix such as:

\[ W = \begin{bmatrix} W_1 & W_2 \end{bmatrix} \quad \text{with} \quad W_1 \in \mathbb{N}^{n_1 m_1} \quad \text{and} \quad W_2 \in \mathbb{N}^{n_2 m_2} \].

So the system (3) is equivalent to:

\[ M_{k+1} = M_k + W_1 \sigma_{k+1}^1 + W_2 \sigma_{k+1}^2 \quad \text{(5)} \]
The reconstruction procedure proposed here is based on writing the system (3) as a descriptor state-space model form. We can consider both the markings of measured places and measured transitions as the outputs of the PN accessible to measurement. We denote $y$ the outputs of the PN. By using the expressions (4) and (5), we obtain the following state equation and output:

$$
\begin{align*}
EM_{k+1} &= AM_k + W_2 \sigma_{k+1}^2 \\
v &= CM_k
\end{align*}
$$

(6)

The generalized state vector and the measurements vector are respectively defined as:

$$
M_k = \begin{bmatrix} M_k \\ \sigma_k^1 \end{bmatrix} \in \mathbb{N}^{n+m_1} \quad \text{and} \quad y_k = \begin{bmatrix} M_k^1 \\ \sigma_k^1 \end{bmatrix} \in \mathbb{N}^{n+m_1}
$$

The matrices $E$, $A$, and $C$ are defined by:

$$
E = [I_n \quad -W_1], A = [I_n \quad 0_{n+m_1}^{m_1}] \quad \text{and} \quad C = \begin{bmatrix} I_{n_1} & 0_{n_1 \times n_2} & 0_{n_1 \times m_1} \\ 0_{m_1 \times n_1} & 0_{m_1 \times n_2} & I_{m_1} \end{bmatrix}
$$

Assumption 1: One assumes throughout this paper that:
All the evolution duration $\tau_i$ (dwell time) of each subsystem of (1) are measurable:

$$
\min \{ \tau_i \} > \tau_{\min} > 0 \quad \text{for} \quad i = 1, \ldots, n
$$

Assumption 1 means that systems with Zeno phenomenon are not considered. Under assumption 1, the global observability of the HDS considered in this paper consists on studying the observability of the PN model (6) associated to the HDS. Therefore we must verify the following theorem:

\textbf{Theorem 1}: The system (6) is observable, if the following classical conditions hold [9], [13]:

\begin{enumerate}
\item[i)] \text{rank} \begin{bmatrix} E \\ C \end{bmatrix} = s, \quad \text{with} \quad s = n + m_1
\item[ii)] \text{rank} \begin{bmatrix} \lambda E - A & W_2 \\ 0 & \lambda I_{m_2} \\ C & 0 \end{bmatrix} = s + m_2, \forall \lambda \in \mathbb{C}, |\lambda| \geq 1
\item[iii)] \text{rank} \begin{bmatrix} E & W_2 \\ C & 0 \end{bmatrix} = s + m_2
\end{enumerate}

The main contribution of this paper is to detect in which configuration the HDS is at a given instant assuming the knowledge of some switching conditions. From these switching conditions, some modes are detected. The modes detected and the known switching’s are defining the output of the discrete model presented by system (6). By assuming that $\sigma_{k+1}^2$ (the unmeasured transitions which present the unknown switching) of the system (6) as an unknown input, we will see in the next section, how to use the discrete system (6) to synthesize an UIO capable of reconstructing $n_2$ unobservable places and $m_2$ unobservable transitions.

\section{ACTIVE MODE DETECTION}

In this section, we will present a new method based on an UIO to detect the active mode and the switching time instant corresponding to the HDS at a given instant where the discrete part is governed by a PN. The proposed observer is designed for the HDS which presented by system (6) in order to estimate both the marking places and the firing vector of transitions by using traditional methods for descriptor state-space systems. Before giving the equation of the observer, the system must verify the observability conditions presented in theorem 1. An UIO for the system (6) is described by the following equation:

$$
\begin{align*}
\dot{z}_{k+1} &= Nz_k + Ly_k \\
\hat{M}_k &= z_k + My_k
\end{align*}
$$

(7)

Where $z_k \in \mathbb{R}^s$, $N$, $L$ and $M$ are matrices of appropriate dimensions that guarantee the asymptotic convergence of the observer.

\textbf{Proposition 1}: There exists an UIO described by (7) for system (6) if there exists a matrix $U \in \mathbb{R}^{s \times m}$ such that:

$$
UE = I_s - MC
$$

(8)

$N$ is a stable matrix

$$
UA = NUE + LC
$$

(9)

$$
UW_2 = 0
$$

(10)

Proof: Let $s + m_1 = s$ (the number of all places and measured transitions). When the observer (7) is applied to the system (6), the estimation error is defined by:

$$
e_k = M_k - \hat{M}_k
$$

$$
e_k = \hat{M}_k - z_k - MC M_k
$$

$$
e_k = (I_s - MC) M_k - z_k
$$

From the first condition of the theorem, there exists a full row rank matrix $[U \ M]$ such that:

$$
[U \ M] \begin{bmatrix} E \\ C \end{bmatrix} = I_s \Rightarrow UE = I_s - MC
$$

The estimation error becomes:

$$
e_{k+1} = UE M_k - z_{k+1}
$$

$$
e_{k+1} = UE M_k - z_{k+1}
$$

$$
e_{k+1} = (UE M_k + W_2 \sigma_{k+1}^2) - NZ_k - LC M_k
$$

$$
e_{k+1} = (UE M_k + W_2 \sigma_{k+1}^2) - NZ_k - LC M_k
$$

$$
e_{k+1} = (UE - NUE - LC) M_k + UW_2 \sigma_{k+1}^2
$$

(12)

The convergence of the estimation error to zero is obtained when: $N$ is a stable matrix, $UA - NUE - LC = 0$ and $UW_2 = 0$. We will present how to calculate the matrices $U$, $N$, $L$ and $M$ satisfying the conditions of the proposition. From (8) and (11):

$$
UE = I_s - MC \Rightarrow [U \ M] \begin{bmatrix} E \\ C \end{bmatrix} = I_s \quad \text{satisfying the conditions of the proposition.}
$$

$$
UW_2 = 0
$$

(13)
The estimation error can be rewritten using (13) and (16) of the form:

$$e_{k+1} = (UA - KC)e_k$$  \hspace{0.5cm} (18)

With K is a matrix gain to be calculated. It is selected such that (N=UA-KC) is stable (i.e. the eigenvalues of the observer are inside the unit circle). Also, it can be calculated if the pair (UA, C) is detectable.

**Theorem 2:** There exists an observer (7) for system (6) if and only if the pair (UA, C) is detectable, i.e.:

$$\begin{align*}
\text{rank} \begin{bmatrix}
\lambda I_s - A & 0 \\
C & 0 \\
\end{bmatrix} = s + m_2, \forall \lambda \in \mathbb{C}, |\lambda| \geq 1
\end{align*}$$

**Proof:** From the second condition of theorem 1, we obtain:

$$\begin{align*}
\text{rank} \begin{bmatrix}
\lambda E - A & W_2 \\
C & 0 \\
\end{bmatrix} = s + m_2, \forall \lambda \in \mathbb{C}, |\lambda| \geq 1
\end{align*}$$

$$\begin{align*}
\text{rank} \begin{bmatrix}
U & 0 & M & 0 \\
0 & I_{n_2} & -\lambda I_p & 0 \\
0 & 0 & 0 & \lambda C \\
0 & 0 & 0 & C \\
\end{bmatrix} = \text{rank} \begin{bmatrix}
\lambda E - A & W_2 \\
0 & I_{n_2} \\
\lambda C & 0 \\
C & 0 \\
\end{bmatrix}
\end{align*}$$

With \(p = n_1 + m_2\)

$$\begin{align*}
\text{rank} \begin{bmatrix}
\lambda U E - U A + \lambda M C & U W_2 \\
0 & I_{n_2} \\
\lambda I_{m_1} & 0 \\
C & 0 \\
\end{bmatrix} = \text{rank} \begin{bmatrix}
\lambda I_s & 0 \\
0 & \lambda I_{m_1} \\
C & 0 \\
\end{bmatrix}
\end{align*}$$

which is equivalent to the detectability of the pair \((UA,C)\).

The theorem (2) implies that the unobservable states \((n_2\text{ places unobservable})\) of the system (6) are stable for each eigenvalue \(\lambda \in \mathbb{C}, |\lambda| \geq 1\). But in this case, the \(n_2\) unmeasured places represents the \(\lambda = 1\) unstable compensation zeros. So, to guarantee a stable observer, we must give an exact state initialization observer (i.e. \(z_0 = U E M_0\)).

Because, the condition (ii) in theorem 1 is not verified and it has \(n_2\) zeros at \(\lambda = 1\).

$$\begin{align*}
\text{rank} \begin{bmatrix}
\lambda E - A & W_2 \\
C & 0 \\
\end{bmatrix}
\end{align*}$$

$$\begin{align*}
\text{rank} \begin{bmatrix}
\lambda I_{m_1} - I_{n_2} & 0 & -\lambda W_{11} & W_2 \\
I_{n_2} & 0 & 0 & 0 \\
0 & 0 & I_{m_1} & 0 \\
\end{bmatrix}
\end{align*}$$

\(=\text{rank} \begin{bmatrix}
0 & 0 & \lambda I_{n_2} & 0 \\
0 & 0 & \lambda I_{m_1} & 0 \\
0 & 0 & 0 & m_1 + m_2 \\
\end{bmatrix}
\end{align*}$$

The subspace corresponding to the eigenvalues \(\lambda = 1\) spanned by \([0 I_s 0 0]^T\) is the unobservable subspace of the pair \((UA, C)\). So, to avoid the instability of the observer, the initial estimation error must be zero at the initial time \((e_0 = U E M_0 - z_0 \Rightarrow z_i = U E M_0)\).

**Remark 1.** In system (6), The global state vector \(M_k\) could not be determined by the initial condition \(M_0\) because we need to know the unmeasured transitions \(\sigma_{k+1}^2\). Therefore, our observer is not a tracker.

Now, we are focusing on the determination of the switching instants of the HDS by estimating the firing vector of transitions composed of a set of measured transitions and a set of unmeasured transitions. Therefore, the unmeasured transitions are calculated in the following theorem as follow:

**Theorem 2:** The unmeasured transitions \(\sigma_{k+1}^2\) are calculated by the following equation:

$$\sigma_{k+1}^2 = W_k^T E M_{k+1} - W_k^T A M_k$$  \hspace{0.5cm} (19)

With \((i)^+ = (i)^T x ((i)^T)^{-1}\).

**Proof:** From the conditions (i) and (iii) of the theorem 1, we conclude that the matrix \(W_k\) is of full column rank. We can therefore estimate the unmeasured transitions from the equation (6) by the equation (19).

**IV. SIMULATION RESULTS**

In this section, we will present some simulation results to demonstrate the effectiveness of the proposed approach. As an example, The PN model corresponding to the discrete part of the HDS is given in Fig. 2.
Fig 2: The PN model

The incidence matrix and initial marking associated with PN model of the HMS are given by:

\[
W = \begin{bmatrix}
-1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & -1 & 1 & -1 & 0 \\
0 & 0 & 1 & -1 & 0 & 1 \\
1 & -1 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 1 & -1
\end{bmatrix}
\quad \text{and} \quad
M_0 = \begin{bmatrix}
0 \\
1 \\
0 \\
0 \\
1 \\
0
\end{bmatrix}
\]

In this system, we consider the places \( p_1, p_2 \) and \( p_5 \) as observables places and transitions \( t_2, t_4 \) and \( t_5 \) as observables transitions (i.e. \( n_1=3, n_2=3, m_1=3 \) and \( m_2=3 \)).

The mathematical model corresponding to the graphical model shown in Fig. 2 is governed by equations of type (6):

\[
\text{rank} \begin{bmatrix} C \\ E \end{bmatrix} = 9, \quad \text{rank} \begin{bmatrix} E \\ W \end{bmatrix} = 12.
\]

Therefore, there exists an UI observer of the form (7). The real mode and the estimate mode of unmeasured places \( p_3, p_4 \) and \( p_6 \) (unobservable modes of the HDS) are shown respectively in Fig.3, Fig.4 and Fig.5. We note that the modes are properly identified.

Finally, from the equation (19), we can obtain the unknown input (unmeasured transitions \( t_1, t_3 \) and \( t_6 \)). The firing vector of transition is arranged as follow:

\[
\begin{bmatrix}
\hat{t}_1 \\
\hat{t}_2 \\
\hat{t}_3 \\
\hat{t}_4 \\
\hat{t}_5 \\
\hat{t}_6
\end{bmatrix} = \begin{bmatrix}
k=0 & k=1 & k=2 & k=3 & k=4 & k=5 & k=6 & k=7 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

V. CONCLUSION

The main contribution of this paper is the detection of the active mode for a HDS governed by a PN in the case where some switching conditions are known. The remaining switching conditions present the unknown inputs for the global system. These unknown switching are activated at unknown instants and it is therefore impossible to locate the active mode of the system. Therefore, an UIO is designed in order to estimate both the marking places and the firing vector of transitions. There are several ways in which this research may be extended. This approach can be used to observe the continuous states and for the design of the control for example.

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