Design and Simulation of Gain Scheduling PID Controller for Ball and Beam System

Bipin Krishna¹, Sagnik Gangopadhay², Jim George³

Abstract—The Ball and Beam system is one of the most enduringly popular and important laboratory models for teaching control systems engineering. It is widely used because of the simplicity to understand as a system, and yet the control techniques that can be studied from it, cover many important classical and modern design methods. Moreover it is an open loop unstable system. This paper presents a gain scheduling PID controller for stabilizing the ball and beam system. The simulation result shows the effectiveness of the proposed method.

Keywords—Ball and beam system (BNB), PID, Gain schedule.

I. INTRODUCTION

THE Ball and Beam is not a model of a real system but it is typical of the dynamics that are found in many of the most challenging areas of modern control[1]. The ball and beam system is one of the most popular and important bench systems for studying control systems. Many classical and modern control methods have been used to stabilize the ball and beam system [2]. Figure-1 shows the ball and beam system (Google Technology, Model GBB1004) which is utilized in this proposed work. The ball and beam system has 2 Degrees-of-Freedom (DOFs). The ball is assumed to have friction, rotary moment of inertial and coriolis acceleration during motion on the beam. It is an open loop unstable, because the system output (the ball position) increases without limit for a fixed input (beam angle). The control job is to automatically regulate the position of the ball by changing the position of the motor [3]. This is a difficult control task because the ball does not stay in one place on the beam, but moves with an acceleration that is proportional to the tilt of the beam.

Due to the nonlinearity and complexity of the governing dynamics, some researchers used non-model based control strategies such as Neural Network, Fuzzy Logic and PID to control the ball position and beam angle. The non-model based method does not require mathematical procedure to derive dynamic equations and to apply linearization. However, these methods are mainly experience-based and cannot guarantee the stability of the system, which may pose challenge to control the unstable ball and beam system.

II. SYSTEM MODELING

Very important application of the ball and beam system is in the field of aerodynamics modeling. The study of the ball and beam system can lead to important conclusions regarding the stability of aircrafts. For e.g. in VTOL (Vertical Take-Off and Landing) aircrafts, regarding the control during take-off or landing, the angle of thrusters jets or diverters must be continually controlled, in order to prevent the aircraft from tipping[4]. These control techniques can be understood better, with the study of ball and beam system. This is one of the examples; however the applications of this system are much wider and can also be applied in fields of chemical process industries and also in power generation.

Figure-1: The Ball and Beam System

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measured positions are fed back to the control system to organize a closed loop control.

The motion of the motor's shaft is governed by IPM100 intelligent drive. This is a high precision, fully digital servo drive with embedded intelligence and 100w power amplifier suitable for brushless/brush motors. Based on feedback information from sensors, it computes and then applies appropriate PWM modulated voltage to the motor windings in such a way that a sufficient torque moves the motor shaft according to the user programmed control algorithm. Figure-2 shows the mechanical schematic of the ball and beam system.

\[
\begin{align*}
\text{Figure-2: Ball and Beam Mechanical System}
\end{align*}
\]

Let the angle between the line that connects the joint of the lever arm with the center of the gear, and the horizontal line be \( \Theta \) (there should be some boundaries on its range so that it can reach the safe maximum and minimum limits); the distance between the center of the gear and the joint of the lever arm be \( d \), and the length of the beam be \( L \). Then the beam angle \( \alpha \) can be expressed in terms of the rotation angle of the gear \( \Theta \) according to the following equation:

\[
\alpha = \frac{d}{L} \Theta
\]

(1)

In turn, as it has just been noted above, the angle \( \Theta \) is connected with the rotational angle of motor shaft through reduction gear ratio \( n=4.28 \).

The controller design task is to keep the position of the ball \( r \) equal to the specified target position by properly manipulating the gear angle \( \Theta \).

The dynamics of the ball is subjected to the gravity, inertial and centrifugal forces. The ball linear acceleration along the beam is given by the following simple equation:

\[
\left( \frac{J}{R^2} + m \right) \ddot{r} + mg \sin \alpha - m \dot{r}^2 = 0
\]

(2)

Where, \( g \) is the gravitational acceleration, \( M \) is the mass of the ball, \( J \) is the ball moment of inertia, \( r \) is the position of the ball along the beam, \( R \) is the radius of the ball.

Here, we assume that the ball rolls without slipping and the friction between the beam and ball is negligible.

Since we are interested in keeping the angle alpha close to 0, we can linearize the above dynamic equation with respect to \( \alpha \) in the neighborhood of zero and then taking into account \( mg \sin \theta = m \ddot{r} \) we get the following linear approximation of the system:

\[
\ddot{r} = \frac{mg}{L\left( \frac{1}{R^2} + m \right)} \Theta
\]

(3)

Where \( x \) is the position of the ball on the beam and \( \ddot{r} \) is the acceleration. The above equation can be used to model the dynamic of the Ball and Beam system.

By taking Laplace transform of equation (3) we get,

\[
R(s) = \frac{mgd}{L\left( \frac{1}{R^2} + m \right)} \times \frac{1}{s^2}
\]

(4)

Where \( R(s) = X(s) \) = position of ball on beam.

\[
\begin{array}{|c|c|c|}
\hline
\text{Symbol} & \text{Quantity} & \text{Value} \\
\hline
\text{g} & \text{Gravity acceleration} & 9.8 \text{ (m/s}^2) \\
\text{m} & \text{Ball mass} & 0.028 \text{ Kg} \\
\text{d} & \text{Distance between the gear center and the moving end of the beam} & 0.04 \text{ m} \\
\text{L} & \text{Length of the beam} & 0.4 \text{ m} \\
\text{R} & \text{Radius of the ball} & 0.01 \text{ m} \\
\text{J} & \text{Ball moment of inertia} & 2\text{m*R}^2/5 \\
\hline
\end{array}
\]

### III. DESIGN METHODOLOGY

In this paper we have proposed an adaptive controller for ball and beam system based on gain scheduling concept. For that we have designed a normal PID controller to stabilize the system and from the performance of this controller an improved gain scheduled PID controller is designed and simulated.

#### A. PID Controller

The PID controller is a well-known industrial feedback control algorithm, which is widely used in industries to control the unstable systems. Figure-3 shows the block diagram of a PID controller which is one of the most powerful but complex controller mode operations combines the proportional, integral, and derivative modes.
The analytical expression of PID controller is given as,

\[ p = K_p e_p + K_p K_i \int_0^t e_p \, dt + K_p K_d \frac{de_p}{dt} + p(0) \]  

(5)

Designing the PID Controller is a crucial step for the final design. This is done to check all the Kp, Ki, Kd values that we will be using for the gain schedule table. First we will be checking and then the set of Kp, Ki, Kd which gives the fastest response will be selected for the table. The beam of the system is with length 390mm. For every 10mm we have found the PID values and this table of PID values are used in gain scheduling design. In table-2 the optimum value of Kp, Ki, Kd are found by trial and error method by observing the settling time. Figure-4 shows the design of the PID controller simulation.

We use the trial and error method to find out the Kp, Ki, Kd values. We put each set of the values to find out the response. The set with the fastest response for that set point is included in the gain schedule table which is the most important part of the final controller. The controller will check every value of the set point entered and then the controller will give us the corresponding control values for the controller. Figure-5 shows the variation of the controller parameters in the gain scheduling table. It can be seen that the Kp value increases steeply but then it stabilizes showing only minor variations within a small range. The Ki value increases steadily, reaches a peak and then decreases more steadily than the increase. The Kd value shows a sinusoidal pattern for the first 6 set points but then stabilizes and shows very less variation. Only towards the end the value keeps on decreasing and comes back to the original starting value.

**B. Adaptive Controller (Gain Scheduling)**

In everyday language, “to adapt” means to change a behavior to conform to new circumstances. Intuitively, an adaptive controller is thus a controller that can modify its behavior in response to changes in the dynamics of the process and the character of disturbances. An adaptive controller is a controller with adjustable parameters and a mechanism for adjusting the parameters. The controller becomes nonlinear because of the parameter adjustment mechanism. It has, however, a very special structure. Since general nonlinear systems are difficult to deal with, it makes sense to consider special classes of nonlinear systems. And adaptive control system can be thought of as having two loops. One loop is a normal feedback with the process and the controller. The other loop is the parameter adjustment loop. In this paper we have considered the gain scheduling concept for the parameter adjustment.

In many situations the dynamics of a process change with the operating conditions of the process. One source of change may be known nonlinearities. It is then possible to change the parameters of the controller by monitoring the operating conditions of the process. This idea is called gain scheduling [6]. It is sometimes possible to find auxiliary variables that correlate well with the changes in process dynamics. It is then possible to reduce the effects of parameter variations simply by changing the parameters of the controller as functions of the auxiliary variables. Figure-6 shows an adaptive system with gain scheduler to change the controller parameter. Gain scheduling can thus be viewed as a feedback control system in...
which the feedback gains are adjusted by using feed-forward compensation. Feed forward controller is effective only when the disturbance is known in the control loop. This disturbance will be controlled by the feed forward compensation. Similarly here the change in set point is known to the control loop and all the PID values are fed in the gain scheduling table. The change in set point is compensated by changing the controller parameter from the table. Figure-6 shows that the controller is getting updated as the set point changes. Gain scheduling has the advantage that the controller parameters can be changed very quickly in response to process changes. Since no estimation of parameters occurs, the limiting factors depend on how quickly the auxiliary measurements respond to process changes.

From the gain scheduling table we have selected the best Kp, Ki, Kd values for a particular range. Table-2 shows the PID controller parameter values for each range in the beam.

<table>
<thead>
<tr>
<th>Range</th>
<th>Kp</th>
<th>Ki</th>
<th>Kd</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 - 50</td>
<td>9.2</td>
<td>2.5</td>
<td>8</td>
</tr>
<tr>
<td>60 - 100</td>
<td>10</td>
<td>2.5</td>
<td>9</td>
</tr>
<tr>
<td>110 - 150</td>
<td>10.2</td>
<td>2.7</td>
<td>9.3</td>
</tr>
<tr>
<td>160 - 200</td>
<td>10.5</td>
<td>3.3</td>
<td>9.3</td>
</tr>
<tr>
<td>210 - 250</td>
<td>10.6</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>260 - 300</td>
<td>10.8</td>
<td>3.2</td>
<td>9.2</td>
</tr>
<tr>
<td>310 - 350</td>
<td>11</td>
<td>2.7</td>
<td>8.7</td>
</tr>
<tr>
<td>360 - 380</td>
<td>10.8</td>
<td>2.1</td>
<td>8.0</td>
</tr>
</tbody>
</table>

IV. RESULTS AND ANALYSIS

Simulation of the designed controller is done using LabVIEW software. The block diagram of the normal PID controller is shown in the figure-4. Response of the PID controller for the standard values taken from the user manual when the set point is given as 1 is shown in figure-7. From this response the settling time of the normal PID controller is observed.

Figure-7: Response of simple PID controller for standard values

Figure-8 shows the block diagram of the control loop with gain scheduling PID.

The gain scheduling PID controller has simulated for various set points and it is observed that the settling time has been reduced to 1.9 seconds from 3.6 seconds (normal PID). The simulation was running in a continuous mode and the settling time is measured manually while we change the set point from one value to a new value. Below figures shows the response of proposed controller for some selected set point values. The values on x-axis in below figures show the simulation time in milliseconds and y axis is the set point given at that particular time. It starts from zero when we start the simulation.
V. CONCLUSION

Ball and beam system is extensively studied and an adaptive controller with gain scheduling concept is designed and simulated. Simulation result shows the accuracy of the gain scheduling controller. Settling time is the parameter considered in this work to measure the performance. From the graphs (9) and (10) we can see that the system is stabilizing faster than the normal PID and settles exactly to the set point. The settling time is reduced to 1.9 seconds. The snap shot of the graphs are taken from when it is in continuous simulation in LabVIEW.

REFERENCES