Abstract—In this paper, a new method for designing backstepping controller in buck/boost chopper is presented. This method is based on appropriate selection of converter’s state variable, which will simplify the designed equations and practical implementation of system. The result of simulation clearly illustrates that the designed controller has zero steady-state error and fast dynamic response and moreover in case of wide variations in load resistance or converter input voltage, the proposed controller can maintain stability.

Keywords—DC to DC converter, Nonlinear and Adaptive Control, Modeling.

I. INTRODUCTION

DC to DC converters are widely used in renewable energy sources, industrial applications such as DC electric motors, computer systems and communication equipment [1]-[4]. According to the non-linear and time-variant nature of these converters, using linear control techniques is based on model linearization. Such model can describe converter’s behavior around the operation point properly. However, despite of feedback design simplicity in linear control, it’s not possible to control the system in wider range. Also presence of large disturbances might have a bad effect on response, even might cause instability [5]. Various non-linear controllers have been proposed to solve this problem. For example sliding mode controller, input/output feedback linearization, passivity based controller and adaptive backstepping [6]-[10].

Adaptive backstepping is presented by P.V.Kokotovic and his colleague in 1992 [10], and is successfully used in lots of non-linear systems, such as electric motors, auto-pilot, submarine, etc. [12]-[14]. This method is based on systematic and recursive design in feedback control of various systems. The most important capability of this technique is its capability to estimating uncertain parameters existing in system’s model. According to the presence of such uncertain parameters in DC to DC converters, using adaptive backstepping method for controlling these converters has been taken into consideration in recent years [15]-[19].

Most of the papers presented in this field are on application of adaptive backstepping controller in buck DC to DC converters. For instance in [15] and [16] this control method is applied in a way that the controller can estimate load resistance. In [17] it is mentioned that more accurate system model can improve response of backstepping controller significantly. For this reason effect of Equivalent Series Resistance (ESR) of inductor is considered. However other parasitic elements such as power switch’s conduction loss has not been considered.

Boost and buck/boost DC to DC converters are more affected by parasitic elements compared with buck converters. This is shown in figure (1). It’s obvious that consideration of these parasitic elements during modeling of buck/boost DC to DC converter can improve the designed controller’s response. However this subject is not mentioned in any of the previous papers. There are at least two main problems in applying adaptive backstepping technique to buck/boost DC to DC converters, considering parasitic elements:

A) Parasitic elements’ value is highly depended on operation point, ambient temperature, etc. and it is impossible to consider a fixed value for parasitic elements of power switches.

B) Adding effect of these elements complicates converter’s state-space model and classic adaptive backstepping controller design – which is used in papers [15]-[20] - is not applicable.

In this paper a new method for applying adaptive backstepping controller in buck/boost DC to DC converter considering parasitic elements effect, is presented. It is shown that by proper modeling, it is possible to estimate converter’s parasitic elements continuously and don’t assign constant value to these time variant elements. Also in designed equations, load resistance is assumed to be uncertain and the controller will estimate its variation. In second section buck/boost DC to DC converter state-space modeling, considering effect of energy dissipating elements is studied. In third section adaptive backstepping controller design is presented in details. Finally in order to investigate response of presented controller, buck/boost DC to DC converter is simulated by MATLAB/Simulink.

II. MODELING OF BUCK/BOOST DC TO DC CONVERTER CONSIDERING POWER LOSS

Generally there are several parasitic elements in DC to DC converter structure. Their presence cause power loss and also...
from control view they may cause change in system behavior. This case in buck/boost DC to DC converter is more important. In figure (1) the response of a buck/boost DC to DC converter, considering the parasitic elements is compared to an ideal one which lossless. It is noticeable that these two characteristics are different specially for high (D) duty cycle values. This figure clearly illustrates necessity for more exact modeling during controller design process.

Power loss in DC to DC has three main sources:

A) Inductor’s loss due to presence of parasitic elements in winding which could be modeled with a resistor($r_L$).

B) Power loss in controlled power switch: according to high frequency nature of DC to DC converters and usual use of power MOSFETs, considering the linear behavior of these controlled switches in triode region, resistance($r_s$) can be used to model it.

C) Voltage drop across the diode in forward bias which is modeled by two segment model.

But of course the ESR of output capacitor is relatively small and will be neglected in this paper. Buck/ boost DC to DC with presence of parasitic elements are shown in figure (2).

During state-space modeling of DC to DC converters, usually inductor’s current and capacitor’s voltage is used as state variables:

$$X = [x_1, x_2]^T = [i_L, v_{out}]^T$$  (1)

When power switch (S) is on, according to equivalent circuit of figure (2-B), state-space equations can be written as following:

$$\dot{X} = A_1 X + B_1$$

$$A_1 = \begin{bmatrix} -\frac{r_s+r_L}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \frac{v_{in}}{L} \\ 0 \end{bmatrix}$$  (2)

And when power switch is off:

$$\dot{X} = A_2 X + B_2$$

$$A_2 = \begin{bmatrix} -\frac{r_D+r_L}{L} & -\frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -\frac{v_{in}}{L} \\ 0 \end{bmatrix}$$  (3)

By combining equations (2) and (3), and considering the relations governing averaged state-space modeling in power electronic converters [1], more accurate buck/boost DC to DC converter can be modeled as below:

$$\dot{X} = AX + B$$

$$A = A_1 u + A_2 (1 - u)$$

$$B = B_1 u + B_2 (1 - u)$$

In this equation, $u$ is duty cycle and usually is considered as control input of system.

During steady-state dynamics of the system is zero and voltage gain of the converter could be formulized as following:

$$\dot{X} = 0 \Rightarrow M = \frac{v_{out}}{v_{in}} = \frac{u - v_{in} (1 - u)}{r_L + r_s u + r_D (1 - u)}$$  (5)

This equation clearly illustrates the influence of each parasitic element on the output voltage of converter. If we assume that parasitic elements in equation (5) are zero, the following equation will be obtained which is completely well-known:

$$M = \frac{u}{1 - u}$$

Main part of loss in DC to DC converters is due to inductor’s ESR. In figure (3) a simple circuit model is used for buck/boost DC to DC converter. In this circuit, only $R_{loss}$ energy dissipating element and power switches are assumed to be ideal. In a similar way averaged state-space model of this converter can be written as below:

$$\dot{X} = \dot{AX} + \dot{B}$$

$$\hat{A} = \begin{bmatrix} -\frac{R_{loss}}{L} & -\frac{1}{L} (1 - u) \\ \frac{1}{C} (1 - u) & -\frac{1}{RC} \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} \frac{v_{out}}{L} \\ v_{in} \end{bmatrix}$$  (6)
\[ \hat{B} = \begin{bmatrix} \frac{V_{in}u}{L} \\ 0 \end{bmatrix} \]

And voltage gain of this one:

\[ \dot{M} = \frac{V_{out}}{V_{in}} = \frac{u}{\frac{R_{loss}}{(1-u)} + (1-u)} \tag{7} \]

If \( \dot{M} = \dot{M} \), it would be possible to assume that these two converters are equivalent and in this case we have:

\[ R_{loss} = \frac{u}{(1-u)[u - \frac{V_{loss}}{V_{in}}(1-u)]} \tag{8} \]

Calculating value of Rloss from equation (8) is not because values of parasitic elements are strongly depended on operating point of power switches, ambient temperature, etc. Moreover, load resistance (R) is an uncertain parameter and because of that adaptive method must be used to estimate Rloss. In this case if Rloss is estimated properly, circuits shown in figures (2) and (3) will be equivalent. According to this, designed controller for equivalent circuit of figure (3) can be used for main circuit of figure (2).

III. CONTROLLER DESIGN

From historical view adaptive backstepping controller has been presented by P.V.Koktovic and his colleague in early 1990 [10]. At first this method was usable only in systems which their state equations form was PPF: Parametric pure form, but gradually developed for PSF: Parametric Strict Form too [12]. Right now this method is usable in broad category which is given relatively comprehensible in [13]. Considering that converters load resistance and \( R_{loss} \) are uncertain, converter’s state space model – which is calculated[3] – are rewritten as below:

\[ \dot{x}_1 = -\frac{1}{L}(\theta_1 x_1 + (1-u)x_2) + \frac{V_{in}}{L} u \tag{9-a} \]

\[ \dot{x}_2 = \frac{V_{loss}}{c_1} (1-u)x_1 - \theta_2 x_2 \tag{9-b} \]

\[ (\theta_1 = R_{loss}, \theta_2 = \frac{1}{R}) \]

If we consider the capacitor’s voltage as an output, the resulted system would have non-minimum phase nature and for this reason adaptive backstepping control method cannot be applied [19]. This problem is easily solved by considering the inductor’s current as an output. We know that controller design purpose is calculation of control input in a way that inductor’s current become equivalent to reference current \( x_2 = I_L(ref) \)

---

**Step one:** according to controlling purpose and above equation, first error variable \( z_2 \) is defined as below:

\[ z_1 = x_1 - I_L(ref) \Rightarrow \dot{z}_1 = \dot{x}_1 - \dot{I}_L(ref) \]

By replacing the equation (9-A) in (10), we have:

\[ \dot{z}_1 = \theta_1 \omega_{11} + I_1 \]

\[ \omega_{11} = -\frac{1}{L} x_1 \]

\[ l_1 = -\frac{1}{L} x_2 (1-u) + \frac{V_{in}}{L} u - I_L(ref) \]  \( \tag{10-C} \)

Since \( \theta_1 \) is uncertain in these equations, it is possible to rewrite (10-A) assuming \( \hat{\theta}_1 \) as an estimated value for this parameter:

\[ \dot{z}_1 = \hat{\theta}_1 \omega_{11} + l_1 + (\theta_1 - \hat{\theta}_1) \omega_{11} \]  \( \tag{11} \)

Now if we choose Lyapunov function as below:

\[ V_1 = \frac{1}{2} \theta_{11}^2 + \frac{1}{2} \gamma_{11}^2 (\theta_1 - \hat{\theta}_1)^2 \tag{12} \]

\( \gamma_{11} \) is a positive coefficient and is called parameter adaption gain. The reason of this naming will be clarified in next steps. Time derivative of Lyapunov function is expressed as below:

\[ \dot{V}_1 = z_1 \dot{z}_1 + \gamma_{11}^2 (\theta_1 - \hat{\theta}_1)(-\dot{\theta}_1) \]  \( \tag{13} \)

With replacing (11) in (13), we have:

\[ \dot{V}_1 = z_1 (\hat{\theta}_1 \omega_{11} + l_1) + (\theta_1 - \hat{\theta}_1) \gamma_{11}^2(-\dot{\theta}_1 + \gamma_{11} x_1 \omega_{11}) \]  \( \tag{14} \)

We know that, the time derivative of Lyapunov function must be negative in order to have a stable designed, this could happen if:

\[ (\hat{\theta}_1 \omega_{11} + l_1) = -c_1 z_1 \]  \( \tag{14-A} \)

\[ -\dot{\theta}_1 + \gamma_{11} x_1 \omega_{11} = 0 \]  \( \tag{14-B} \)

Which, in equation (14-A), \( c_1 \) is a constant and positive coefficient.

**Step two:** generally equation (14-A) might not be correct, and the difference between two sides of this equation could be defined as below:

\[ z_2 = \hat{\theta}_1 \omega_{11} + l_1 - (-c_1 z_1) = (\hat{\theta}_1 \omega_{11} + l_1) + c_1 z_1 \]  \( \tag{15} \)

By replacing equations (10),(10-B) and (10-C) in equation (15) we will have:

\[ z_2 = -\dot{\theta}_1 \frac{1}{L} x_1 - \frac{1}{L} x_2 (1-u) + \frac{V_{in}}{L} u - \dot{I}_L(ref) + c_1 x_1 - c_1 l_1 (ref) \]  \( \tag{16} \)

Time derivative of \( z_2 \) is expressed as below. It worth noting that while simplifying the following equation, time derivatives of variables \( x_1 \) and \( x_2 \) are replaced from equations (9-A) and (9-B):

\[ \dot{z}_2 = \theta_1 \left[ \hat{\theta}_1 \frac{1}{L} x_1 - c_1 \frac{1}{L} x_1 \right] + \theta_2 \left[ \frac{1}{L} x_2 (1-u) - \frac{V_{in}}{L} u - \dot{I}_L(ref) - \frac{1}{L} x_1 (1-u)^2 + \frac{1}{L} x_2 u + \frac{V_{in}}{L} \dot{u} - \dot{I}_L(ref) - c_1 \frac{1}{L} x_2 (1-u) + c_1 \frac{V_{in}}{L} u - c_1 \dot{l}_1 (ref) \right] \]

We can simplify this equation as below:

\[ \dot{z}_2 = \theta_1 \omega_{112} + \theta_2 \omega_{22} + I_2 \]  \( \tag{18-A} \)

\[ \omega_{112} = \hat{\theta}_1 \frac{1}{L} x_1 - c_1 \frac{1}{L} x_1 \]  \( \tag{18-B} \)
\[ \omega_{22} = \frac{1}{L} x_2 (1 - u) \]  
\[ l_2 = -\frac{\theta_1}{L} x_1 + \frac{1}{L} x_2 (1 - u) - \frac{v_{in}}{L^2} u - \frac{1}{L C} x_1 (1 - u)^2 + \frac{v_{in}}{L} u + \frac{v_{in}}{L} u - \dot{i}_L (\text{ref}) - c_1 \frac{1}{L} x_2 (1 - u) + c_1 \frac{v_{in}}{L} u - c_1 \dot{i}_L (\text{ref}) \]  
\[ \]  
We rewrite the time derivative of \( z_2 \) by using equation (18-A) as below:
\[ \dot{z}_2 = \theta_1 \omega_{12} + \theta_2 \omega_{22} + l_2 + (\theta_1 - \theta_1) \omega_{12} + (\theta_2 - \theta_2) \omega_{22} \]  
In equation above \( \dot{z}_2 \) is estimated value for \( \theta_1 \). Now in this step we should choose Lyapunov function:
\[ V_2 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} \gamma_1^1 (\theta_1 - \theta_1)^2 + \frac{1}{2} \gamma_2^2 (\theta_2 - \theta_2)^2 \]
It’s time derivative is:
\[ \dot{V}_2 = z_1 \dot{z}_1 + z_2 \dot{z}_2 + \gamma_1^1 (\theta_1 - \theta_1) (\dot{\theta}_1) + \gamma_2^2 (\theta_2 - \theta_2) (\dot{\theta}_2) \]
\[ \]  
By considering equations (11) and (15) simultaneously, time derivative of \( z_1 \) can be written as below:
\[ \dot{z}_1 = -c_1 z_1 + z_2 + (\theta_1 - \theta_1) \omega_{11} \]
By replacing \( Z_1 \) and \( Z_2 \) respectively from equations (19) and (22) in time derivative of Lyapunov function (equation (21)) and considering equations (10-B), (18-B) and (18-D), we have:
\[ \dot{V}_2 = -c_1 z_1^2 + z_1 z_2 + z_2 [\frac{1}{L} \dot{\theta}_1 x_1 + \frac{1}{L^2} x_2 (1 - u) - \frac{v_{in}}{L^2} u] - \frac{1}{L} x_1 (1 - u)^2 + \frac{v_{in}}{L} u + \frac{v_{in}}{L} u - \dot{i}_L (\text{ref}) - c_1 \frac{1}{L} x_2 (1 - u) + c_1 \frac{v_{in}}{L} u - c_1 \dot{i}_L (\text{ref}) + \frac{1}{L} \dot{x}_1 - c_1 \frac{1}{L} \dot{x}_1 + \frac{1}{L} \dot{x}_2 (1 - u) + (\theta_1 - \theta_1) \gamma_1^1 (\dot{\theta}_1) + \gamma_2^2 (\theta_2 - \theta_2) \gamma_2^2 (\dot{\theta}_2) \]  
In the above equation if we assume that the coefficient of \( z_2 \) equals to \(-c_2 z_2\) and also assume that the coefficients of \( (\theta_1 - \theta_1) \gamma_1^1 \) and \( (\theta_2 - \theta_2) \gamma_2^2 \) is zero:
\[ V_2 = -c_1 z_1^2 + z_1 z_2 + c_2 z_2^2 \]  
In the above equations \( c_2 \) is a constant and positive coefficient. It can be seen that if \( 4 c_1 c_2 > 1 \), equation (24) is always negative and stability of the system is guaranteed. Buck/boost DC to DC converter’s control input is obtained by assuming that coefficient of \( z_2 \) is equals to \(-c_2 z_2\) in equation (23):
\[ \dot{u} = \frac{L}{x_2 + v_{in}} \left\{ -\frac{1}{L} x_1 + \frac{1}{L^2} x_2 (1 - u) - \frac{v_{in}}{L} u - c_1 \frac{1}{L} x_1 + c_1 \frac{v_{in}}{L} u + c_1 \frac{v_{in}}{L} u - c_1 \dot{i}_L (\text{ref}) - \dot{i}_L (\text{ref}) \right\} \]
\[ \]  
Similarly, if we consider that the coefficient of the expressions \( (\theta_1 - \theta_1) \gamma_1^1 \) and \( (\theta_2 - \theta_2) \gamma_2^2 \) is zero in (23), we can obtain estimation rules for uncertain parameters. During simplification, equations (10-B), (18-B) and (18-C) are used.
\[ \dot{\theta}_1 = \gamma_1^1 [\frac{1}{L} x_1 + \frac{1}{L^2} x_2 (1 - u)] \]
\[ \dot{\theta}_2 = \gamma_2^2 [\frac{1}{L} x_2 (1 - u)] \]

**IV. SIMULATION RESULTS**

In this part buck/boost DC to DC converter based on proposed adaptive backstepping controller is simulated by MATLAB/Simulink. General block diagram of the system is shown in figure (4). Also simulation parameters and other used elements are summarized in table (1). Choosing these parameters are relatively straightforward and are given in details in [1]. The response of designed controller to step reference is shown in figure (5). It’s obvious that this method has fast dynamic response and also has zero steady-state error. The response of the controller to load variation is shown in figure (6). It’s obvious that adaptive controller is completely robust.

**Table (1): simulation parameters**

<table>
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<th>Parameter</th>
<th>Value</th>
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<td>c1</td>
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<tr>
<td>c2</td>
<td>25000*0.25</td>
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<tr>
<td>C</td>
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<td>L</td>
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<td>teta2=1/R</td>
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<tr>
<td>Vin</td>
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<td>gamma1=1e-7</td>
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<td>gamma2=1e-7</td>
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**V. CONCLUSION**

A novel adaptive backstepping controller based on accurate averaged state-space model is proposed for buck/boost DC to DC converter to regulate the inductor current. In the design procedure load resistance and all of the parasitic elements assumed to be uncertain. Finally in order to verify accuracy of the proposed controller, buck/boost converter is simulated based on the designed controller via MATLAB/Simulink. Results clearly show that, in spite of large variations of load, the controller has good steady-state and dynamic response.
IV. ACKNOWLEDGMENT

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REFERENCES

[9] Siew-Chong Tan, Member, IEEE, Y. M. Lai, Member, IEEE, and Chi K. Tse, Fellow, IEEE, "Indirect Sliding Mode Control of Power Converters Via Double Integral Sliding Surface" IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL. 23, NO. 2, MARCH 2008