Length Optimization and Pulse Compression in Large Signal Modeling of Pulsed Pump Silicon Raman Amplifiers


Abstract— In this paper, large signal modeling of silicon raman amplifiers with the pulsed pump is presented. In long amplifiers, pump power is depleted due to nonlinear absorption and gain enhancement. Therefore, signal power is comparable with pump power and small signal modeling is not valid. It is shown that when the length of amplifier increases above the optimized length, gain is reduced. Also, the optimized length of amplifier versus pump intensity is presented. Finally, pulse compression using silicon raman amplification is fulfilled.

Keywords— Free carrier absorption, Pulse compression, Silicon raman amplifier, Two photon absorption.

I. INTRODUCTION

Silicon has been considered as the main material in microelectronic industry. Optical characteristics of silicon including light guiding, light modulation and light emission, led to the widespread applications in optical industry, especially in telecommunications.

Silicon on insulator (SOI) has been known as the main platform in fulfillment of electronic and photonic systems. Most interesting feature of SOI is the possibility of integration of optical and electronic devices, simultaneously[2].

Nonlinear absorption is known as the main disadvantage of SOI. On the other hand, unique specifications of these waveguides, including high refractive index and large third order nonlinearities led to the widespread application in the optical telecommunications and other optical systems[3].

So far, large signal modeling is not studied comprehensive, in silicon raman amplifiers. Rukhlenko et al. have been investigated, large signal modeling of silicon raman amplifiers with continues wave (CW) pumping[4]. In their modeling they have shown, it is possible to provide an analytically solutions for the pump and signal intensities during the waveguide. However, since they have neglected two photon absorption (TPA) effects, their solutions differ significantly from precise solutions.

However, pulse compression is possible only in pulsed pump scheme. So, in this paper modeling of large signal and length optimization of pulsed pump silicon raman amplifiers are presented. In the sequence, it is shown, in the long amplifiers, small signal modeling is not valid and large signal modeling must be considered. In the pulsed pump raman amplifiers, it is impossible to neglect effect of TPA. Free carrier absorption (FCA) density, is also time dependence. So, it is difficult to demonstrate an analytical solution for the pump and signal intensities.

The remainder of this paper is organized as follows. Section II describes the long signal modeling theory. Results are presented in section III. A Brief summary is given in section IV.

II. THEORY

In order to express appropriate statements for the signal and pump intensities, equations of Rukhlenko et al. have been used. They have shown pump and signal intensities at the stocks frequency, can be expressed by two coupled equations as follows[4].

\[
\frac{dI_p}{dz} = -\alpha_p I_p - \beta_p I_p^2 - (2\beta_{ps} + g_r)I_p I_s + f_1(I_p, I_s)
\]

\[
\frac{dI_s}{dz} = -\alpha_s I_s - \beta_s I_s^2 - \frac{\alpha_p}{\alpha_p} (2\beta_{ps} - g_r)I_p I_s + f_2(I_p, I_s)
\]

In these equations, \(I_p\) and \(I_s\) are the pump and signal intensities, \(\alpha_p\) and \(\alpha_s\) are the angular frequencies of pump and signal, \(\beta_p\) and \(\beta_s\) are the self TPA coefficients at the pump and signal frequencies and \(\beta_{ps}\) is the cross TPA coefficient between pump and signal. Also, \(g_r\) is the raman gain coefficient.

Linear absorption coefficients are almost frequency independent and are shown with \(\alpha\). Self TPA coefficients also
are frequency independent.

Cross TPA coefficient approximately is equal to self TPA coefficients and they are shown with $\beta$. Other terms are related to the free carrier absorption densities, and can be expressed as the following equations:\[5\].

$$f_1(I_p, I_s) = -\sigma_p NI_p$$ \hspace{1cm} (3)

$$f_2(I_p, I_s) = -\sigma_s NI_s$$ \hspace{1cm} (4)

$\sigma_p$ and $\sigma_s$ are the free carrier absorption cross sections at the pump and signal wavelengths. Empirical quantities for these coefficients are expressed as follows.

$$\sigma_p = \sigma_0 \left(\frac{\lambda_p}{1550\text{nm}}\right)^2$$ \hspace{1cm} (5)

$$\sigma_s = \sigma_0 \left(\frac{\lambda_s}{1550\text{nm}}\right)^2$$ \hspace{1cm} (6)

$\sigma_0$ is free carrier absorption cross section at the wavelength of 1550 nm. $\lambda_p$ and $\lambda_s$ are the wavelengths of pump and signal. Also $N$ is the free carrier absorption density and it is time dependence in pulsed pump scheme\[6\].

$$\frac{dN}{dt} = (\rho_p I_p^2 + \rho_s I_s^2 + \rho_{ps} I_p I_s) - \frac{N}{\tau}$$ \hspace{1cm} (7)

$\tau$ is the effective free carrier lifetime in silicon. Also, $\nu_p$ and $\nu_s$ are the pump and signal frequencies and $h$ is Planck constant.

According to these equations, pump and signal intensities in the large signal modeling of pulsed pump silicon raman amplifiers can be expressed as follows.

$$\frac{dI_p}{dz} = -a I_p - \beta I_p^2 - (2\beta - g_r)I_p I_s - \sigma_p NI_p$$ \hspace{1cm} (11)

$$\frac{dI_s}{dz} = -a I_s - \beta I_s^2 - \frac{\omega_s}{\omega_p}(2\beta - g_r)I_p I_s - \sigma_s NI_s$$ \hspace{1cm} (12)

Any analytical solution cannot be found for these equations, and they must be solved numerically. For the small signal modeling with the assumption of $I_s<<I_p$, equations of $I_p$, $I_s$ and $N$ is reduced to the following equations.

$$\frac{dI_p}{dz} = -a I_p - \beta I_p^2 - \sigma_p NI_p$$ \hspace{1cm} (13)

$$\frac{dI_s}{dz} = -a I_s - \frac{\omega_s}{\omega_p}(2\beta - g_r)I_p I_s - \sigma_s NI_s$$ \hspace{1cm} (14)

$$\frac{dN}{dt} = (\rho_p I_p^2) - \frac{N}{\tau}$$ \hspace{1cm} (15)

Which are similar to\[2\]. But the second term at the right side of (14) is multiplied by the ratio of signal and pump frequencies. To have a sufficient accuracy, effect of this ratio also is considered. Amplifier gain can be calculated from the following equation, where $L$ is the amplifier length\[7\].

$$G = 10 \log \left[ \frac{I_s(L)}{I_s(0)} \right]$$ \hspace{1cm} (16)

In the small signal modeling, signal intensity evolution can be expressed by the following equation.

$$\frac{dI_s}{dz} = \left[ -\alpha - \frac{\omega_s}{\omega_p}(2\beta - g_r)I_p - \sigma_s N \right]$$ \hspace{1cm} (17)

Since the right hand side of equation is independence from $I_s$, the amplifier gain can be estimated by following equation.

$$G = 4.343 \int_0^L \left[ -\alpha - \frac{\omega_s}{\omega_p}(2\beta - g_r)I_p - \sigma_s N \right] dz$$ \hspace{1cm} (18)

Unlike the large signal modeling, in the small signal modeling, amplifier gain is independent of initial signal intensity. So in the large signal modeling, the effect of the signal initial intensity must be considered.

### III. RESULTS

#### A. Large signal gain

In order to solve the coupled equations, parameters of practical amplifiers must be considered. So, the reported quantities for pulsed pump is used which are shown in the following table\[5\].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.22 dB/cm</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5 cm/Gw</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>1.45×10^{-17} cm^2</td>
</tr>
<tr>
<td>$g_r$</td>
<td>10.5 cm/Gw</td>
</tr>
</tbody>
</table>
Effective lifetime of free carriers also is dependent from the waveguide dimensions and different values in the range of 16ns[8] up to 25ns[7] are reported for this parameter in silicon. For gain enhancement, effective lifetime must be reduced. So, the value of 16 ns is picked out. Also, the common values for pump, signal and amplifier characteristics are used and is shown in Table 2 [4],[7].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_p(0))</td>
<td>(I_p \exp(-4Ln2t^2/T^2))</td>
</tr>
<tr>
<td>(I_s(0))</td>
<td>0.01(I_p)</td>
</tr>
<tr>
<td>(L)</td>
<td>4.8cm</td>
</tr>
<tr>
<td>(I_o)</td>
<td>50 MW/cm²</td>
</tr>
<tr>
<td>(T_R)</td>
<td>17 ns</td>
</tr>
</tbody>
</table>

\(T_R\) is full width at half maximum (FWHM) of the pump intensity. Stocks shift of silicon is 15.6 THz. Pump frequency is assumed to be 1427 nm which leads to the signal amplification frequency of 1541.1 nm.

Fig.1 shows the gain of amplifier. As it can be seen, the gain of amplifier is time dependent. This leads to the time dependence of signal in the output of amplifier, despite of constant signal in the input. In this figure, maximum gain is called effective gain and in the remainder, instead of the time dependent gain, effective gain is considered.

Comparing the results of small signal and large signal modeling, it can be concluded that the gain of the amplifier is greater in the small signal modeling. But, due to short length of amplifier, this difference is negligible. However in large amplifiers, large signal modeling must be considered. So far, widespread range of \(T_R\) from 30 ps[9] up to 17 ns[5] has been reported. Nowadays, manufacturing of ultra-short pulses in the range of femtoseconds is accessible. So, the effect of pulse width on the amplifier gain must be considered.

Fig.2 illustrates the effective gain versus \(T_R\) and for some quantities of \(I_o\). These quantities for maximum pump intensities are common quantities where reported in experimental papers. Also, variation range of pulse width is covering the reported data in these experiences.

With the reduction of \(T_R\), amplifier gain is improved. But, since the generation of ultra-short pulses is so difficult, pulse width of pump is chosen to be at least 10 ps. This figure also shows that the effect of \(T_R\) on the amplifier gain for the high intensities is much greater than low intensities.

The length of most practical silicon raman amplifiers, has been limited to a few centimeters. In the short amplifiers, pump depletion is negligible and small signal modeling is reliable. But, in the large amplifiers, because of pump depletion due to TPA and FCA, small signal modeling is not valid. In order to examine this issue, large signal and small signal gain of amplifier has been investigated. Fig.3 and Fig.4 show the gain of amplifier versus amplifier length.

In both small signal modeling and large signal modeling, while the amplifier length increases above the specified length, amplifier gain is reduced. This length is called optimized length. Already, this phenomenon has been observed in the CW pumping scheme[4].
Large signal modeling, leads to the gain suppression and optimized length reduction, too. In the high pump intensities, this reduction is more obvious than low intensities.

B. Length optimization

For all of pump intensities, always an optimized length is existed. So, it is needed to understand the optimized length for different intensities. This procedure is called length optimization of amplifier.

Assuming $I_s(0)=0.01I_0$, optimized length is shown in Fig.5 for various pulse widths.

In practice, the signal intensity is constant at the beginning of amplifier. So, the results must be shown for constant input signal intensity. Since common values for maximum pump intensities are in the range of 30 up to 50 MW/cm$^2$, input signal intensity is assumed to be 0.4 MW/cm$^2$ and results are shown in Fig.6 for various pulse widths.

Considering these results it is concluded that for the pulse widths lower than 100 ps, the optimized length is approximately independent of pulse width.

Since for the pump widths lower than 100 ps, optimized length is always the same, it is an advantage in the design of silicon raman amplifiers.

C. Pulse compression

In addition of signal amplification, silicon raman amplifiers can be used as pulse compressor. Until now, amplifiers with time independence input signal, have been studied. But, in the pulsed signal amplifiers with the pump width less than signal width, signal can be compressed.

In fact, when the pump intensity is existed, signal intensity is amplified due to raman amplification and in the absense of pump intensity, signal intensity is attenuated due to nonlinear absorption. So, signal compression is accessible.

It has been observed that pump width reduction bellow a few picoseconds, especially in low intensities, cannot help to gain enhancement. But in order to signal compression, ultra short pulses must be used. Already, 1 fs compressed pulse in silicon is obtained from 30 fs input signal. Pulse compression is fullfilled by self phase modulation (SPM) and group velocity dispersion (GVD) effects [11]. If the pump width of 1 fs is used, similar compression can be yield in silicon raman amplifiers, too.

The quantities of Table 3 is used for signal compression in contrast with [11].

### Table 3
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
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<tbody>
<tr>
<td>$I_s(0)$</td>
<td>$I_0 \exp\left(-\frac{t^2}{T_s}\right)$</td>
</tr>
<tr>
<td>$I_0$</td>
<td>$0.01I_0 \exp\left[-\frac{2t^2}{(30T_s)^2}\right]$</td>
</tr>
<tr>
<td>$L_{\text{opt}}$</td>
<td>21.4 cm</td>
</tr>
<tr>
<td>$I_0$</td>
<td>30 MW/cm$^2$</td>
</tr>
<tr>
<td>$T_s$</td>
<td>1 fs</td>
</tr>
</tbody>
</table>

Input signal intensity and output signal intensity at optimized length and half of optimized length is shown in Fig.7.
as can be seen, pulse compression occur, even in the amplifiers shorter than optimized length. But, in order to output signal be close to ideal condition, amplifier length must be close to optimized length.

![Graph showing input and output signal intensities](image)

**Fig.7 Input signal intensity and output signal intensity**

**IV. CONCLUSION**

In this paper, large signal modeling of silicon raman amplifiers with pulsed pump was performed. It was shown in large amplifiers; small signal modeling cannot describe the amplifier gain. So, in silicon raman amplifiers, the most important reason of pump depletion is TPA and FCA. Unlike the silica raman amplifiers, which pump depletion is caused by gain enhancement. So, in the silicon raman amplifiers, large signal modeling is more important than silica raman amplifiers. Also, it was shown for all pump intensities, an optimized length is existed. It was observed that for the narrow pump pulses, optimized length is almost pulse width independent. Finally, pulse compression is done using raman silicon amplification.

**REFERENCES**


