Regional Flood Frequency Analysis - Luanhe basin – Hebei - China

Badreldin G. H. Hassan and Prof. Feng Ping

Abstract—The design value to be used in the design of hydraulic structures is founded by using a statistical method called the frequency analysis. Flood frequency analysis is essentially a problem of information scarcity in arid and semi-arid regions. Practically in these regions, the length of records is usually too short to insure reliable quantile estimates. Regional flood frequency (RFFA) analysis involves two majors’ steps: (1) Grouping of sites into homogeneous regions, and (2) Regional estimation of flood quantiles at the site of interest.

The main objectives of this study were to estimate regionalized parameters and identification of the best-fit distribution by L-moment approach in seventeen selected stations in Luanhe basin where the parameters of the distributions are found by using L-Moments technique. This study includes information about L-moments and regional frequency analysis, applications result and discussion of the results. The regionalization technique used is the index flood procedure; the accuracy of the estimated quantiles is obtained by using the Monte Carlo simulation. Discordancy and homogeneity tests were done by L-moments based measures. In this study, five three parameter distributions generalized logistic (GLO), generalized extreme-value (GEV), generalized normal (GNO), Pearson type-3 (PE III) and generalized pareto distributions (GPA) were fitted to the four homogeneous regions. The three parameters of these distributions are estimated by the L-moment approach. The study area was divided into four regions according to Ward’s methods.

Keywords—3-days extreme flood, Flood frequency analysis, Index flood, L-moments, Regional analysis.

I. INTRODUCTION

Extreme high precipitation amounts are among environmental events with high flood amounts are the most disastrous consequences for human society. Estimates of their return periods and design values are of great importance in hydrologic modelling, engineering practice for water resources and reservoirs design and management, planning for weather-related emergencies, etc.

Peak or flood flow is an important hydrologic parameter in the determination of flood risk, management of water resources and design of hydraulic structures such as dams, spillways, culverts and irrigation ditches. Estimation must be fairly accurate to avoid excessive costs in the case of overestimation of the flood magnitude or excessive damage and even loss of human lives while underestimating the flood potential. Thus, it is fundamental to estimate how often a specific flood event will occur, or how large a flood will be for a particular probability of exceedence or recurrence interval. This might be achieved through at-site or regional flood frequency analysis procedures.

When the catchment of interest is either ungauged or the stream flow record is of short duration, practitioners resort often to the transfer of flood information from gauged to ungauged sites. In this case, floods are best estimated using regional flood frequency analysis, within a pre-specified hydrologically homogeneous region, overcoming thereby the limits on transferability of the flood design data. Regional flood frequency analysis was shown to yield more accurate quantile estimates than single-site frequency analysis even for moderately heterogeneous regions (Hosking and Wallis 1988 and Potter and Lettenmaier 1990).

Flood frequency analysis plays an important role in hydrologic and economic evaluation of water resources projects. It helps to estimate the return periods and their corresponding event magnitudes, thereby creating reasonable design criteria. The basic problem in flood studies is an information problem which can be approached through frequency analysis. The classical approach to flood frequency analysis is hampered by insufficient gauging network and insufficient data, especially when the interest is in estimating events of large return periods. At-site flood frequency analysis is the analysis in which only flood records from the subject site are used. More commonly, it will be necessary to carry out a regional analysis where flood records from a group of similar catchments are used. Regionalization or regional analyses are thought to compensate for the lack of temporal data.

One of the initial steps in regional flood frequency analysis involves identifying homogeneous regions. Regions are subsets of the entire collection of sites and consist of catchments at which extreme flow information is available or for which estimates extreme flow quantiles is required. A region can be considered to comprise a group of sites from which extreme flow information can be combined for improving the estimation of extreme quantile at any site in the region. The identification of an appropriate group of side is an important step in regional frequency analysis.

The L-moment based method of the regional frequency analysis of extremes 3-days important to being utilized for
the area of Luanhe basin; maximum 3-days flood amounts over 1932-1970 measured at 17 stations are used as an input dataset. The first step of the regional analysis consists in an identification of homogeneous regions. Candidate regions were formed by the cluster analysis of site characteristics (including longitude, latitude, elevation, and mean annual precipitation), using the Ward’s method. The final homogeneous regions enter the next steps of the regional frequency analysis which concern selection of the most appropriate distribution, and estimation of parameters and quantiles of the fitted distribution together with their uncertainty.

II. REGIONAL FREQUENCY ANALYSIS BASED ON L-MOMENTS

A. Regional frequency analysis

The design of water resources projects requires a design criterion expected to be observed for a given return period. As a statistical method, frequency analysis is used to estimate how often a specified event will occur. Frequency analysis is an efficient tool in design which reduce the cost of the project due to employment of the forecasting technique.

In a regional frequency analysis, data from several sites are used in estimating frequencies at any one site. Regional frequency analysis aims to overcome sampling variability associated with short record lengths by “trading space for time,” i.e., using data from nearby or comparable sites to derive drought frequency estimates for any given site in a homogeneous region (Stedinger et al., 1993).

The advantage of the regional over ‘at-site’ estimation is greater at distribution tails which are focused by practical applications. Many methods recommended by national organizations for general use by hydrologists have a regional component. Different methods of regional frequency analysis were reviewed by Cunnane (1988) who rated the algorithm based on probability-weighted-moments (PWMs; Greenwood et al., 1979) as the best. L-moments are statistical quantities that are derived from PWMs and increase the accuracy and ease of use of the PWM-based analysis (Hosking and Wallis, 1997).

B. L-moments

L-moments are a recent development in mathematical statistics which facilitates the estimation process in the frequency analysis; they represent an alternative set of scale and shape statistics of a data sample or a probability distribution. Their main advantages over conventional (product) moments are that they are able to characterize a wider range of distributions, and (when estimated from a sample) are less subject to bias in estimation and more robust to the presence of outliers in the data (e.g. Royston, 1992; Sankarasubramanian and Srinivasan, 1999; Ulrych et al., 2000). The latter is because ordinary moments (unlike L-moments) require involution of the data which causes disproportionate weight to be given to the outlying values. The identification of a distribution from which the sample was drawn is more easily achieved (particularly for skewed distributions) using L-moments than conventional moments (Hosking, 1990). The method of L-moments is also more efficient in estimating parameters of a fitted distribution compared to the maximum likelihood method (Hosking et al., 1985). (See Appendix I for a formal definition of L-moments.)

L-moments may be applied in four steps of the regional frequency analysis (Hosking and Wallis, 1997; Alila, 1999; Adamowski, 2000):

i. Screening of the data. L-moments are used to construct a discordancy measure which identifies unusual sites with sample L-moment ratios markedly different from the other sites. These unusual sites merit close examination.

ii. Identification of homogeneous regions. L-moments are used to construct a summary statistics in testing heterogeneity of a region.

iii. Choice of a frequency distribution. L-moment ratio diagram and/or regional average L-moments are used in testing whether a candidate distribution gives a good fit to the region’s data.

iv. Estimation of the frequency distribution. Regional L-moments are used to estimate parameters of the chosen distribution.

The L-moment based methods of regional frequency analysis are now being adopted by many organizations worldwide (Hosking and Wallis, 1997). Recent findings indicate that appropriate modifications of the ‘standard’ procedure can improve its performance and the reliability of design values (e.g. Sveinssson et al., 2001). Future directions and challenges in the regional analysis involve development of a more statistical methodology, explanation of the superiority of L-moments for small samples, incorporation of covariates into regional extreme value models, and dealing with the spatial dependence of extremes.

III. DATA

Extreme maximum 3-days flood amounts measured at 17 stations in Luanhe basin (area of 319000 square km, this is divided into two basins – jidong yan hai basin with area of 54400 square km, and tuhai majia he basin with area of 264600 square km), with altitudes ranging from 32 to 1166 m a.s.l., (Figure 1). The data span the period of 1930-1970.

The study area refers to Luanhe basin located in Hebei-China, with the geographic coordinates from 112o ~ 120o East longitude and 35o ~ 43o North latitude. The hydrological and climatic data such as maximum 3-days annual flood series were obtained from Tianjin University-School of Civil Engineering-Department of Hydrology and water resources. Most of the gauging stations in this part have short data series with a large number of missing data. The selection of stations in the basin is made such that at least 7 years of historical flood data are available. The average record length for the stations was 15 years with range from 7 to 33 years. After the preliminary screening of the sites 17 gauging station were
selected for this study. (See the geographic and hydrological characteristic in tables 1. (a)&1. (b)).

IV. APPLICATION

A. Identification of Homogeneous Regions using Cluster Analysis

In this study, determination of homogeneous regions is done by Ward’s clustering method as a much known approach for determination of homogeneous regions in regional frequency analysis. The physical characteristics such as the area, longitude, latitude, and the elevations of selected stations in the basin, was subjected to hierarchical clustering based on the Ward’s method using Euclidean distance.

B. Identification of the best-fit distribution

In general flood frequency analysis a single frequency distribution is fitted to the data from several sites and the aim is therefore not to identify a ‘true’ distribution but to find a distribution that will yield more accurate quantile estimates for each site. Firstly, in this study, five three parameter distributions were selected, and fitted to the four homogeneous regions; generalized logistic (GLO), generalized extreme-value (GEV), generalized normal (GNO), Pearson type-3 (PE III) and generalized pareto distributions (GPA).

C. Estimation of distribution parameters using L-moments

In the L-moments approach, the three parameters (location,scale,shape) of each probability distribution in regiona flood frequency analysis are obtained by the regional average of the L-moment and L-moment ratios. These parameters are needed for estimating the peak flow of each recurrence interval of interest by the selected the best-fit distribution. The values of the parameters depend on the shape of density and cumulative functions of different distributions and their parameter estimation formulas. (See table 2)

D. GOODNESS-OF-FIT TEST

The goodness-of-fit measure, Z, judges how well the simulated L-skewness and L-kurtosis of a fitted distribution matches the regional average L-skewness and L-kurtosis values obtained from the observed data. For a selected distribution, Z is defined by Hosking and Wallis (1993) as:

\[ Z_{dist} = \left( \frac{\tau_4^{dist} - \tau_4 + \beta_4}{\sigma_4} \right) \]

(2)

Where \( \tau_4 \) is the average L-kurtosis value computed from the data of a given region; \( \tau_4^{dist} \) is the average L-kurtosis value computed from simulation for a fitted distribution; \( \beta_4 \) is the bias of \( \tau_4 \); and \( \sigma_4 \) is the standard deviation of L-kurtosis values obtained from simulation. Thus:

\[ \beta_4 = \left( N_{sim} \right)^{-1} \sum_{m=1}^{N_{sim}} \left( \tau_4^m - \bar{\tau}_4 \right) \]

(2)

where \( \bar{\tau}_4^m \) is the regional average L-kurtosis and is to be calculated for the nth simulated region. Then, a given distribution is declared to be of adequate fit if \( Z_{dist} \) is sufficiently close to zero. A reasonable criterion is \( \left| Z_{dist} \right| \leq 1.64 \). This criterion corresponds to acceptance of the hypothesized distribution at a confidence level of 90% and shows approximately a standard normal distribution if the at-site L-kurtosis statistics have independent identical normal distributions.

V. RESULTS

A. Screening of the data

The sites that are grossly discordant with the group as a whole were identified using the discordancy measure (D) based on L-moments. The formal definition of the discordancy measure can be found in Hosking and Wallis (1993) (See the appendixII); it yields a value of D for each measuring site. Critical values for the discordancy statistic are tabulated; for the number of sites \( \geq 15 \), the critical value is 3.

The largest Di is founded 4.69 for site 27; and there is only one site where the Di value exceeds the critical value. Therefore, the site either not belong to the region or there is an evidence of gross error in the data.

B. Identification of regions

The formation of regions was based on the cluster analysis of four ‘site characteristics’: longitude, latitude, elevation, and the area. Using ‘at-site statistics’ (quantities calculated from the at-site values of the analyzed variables) instead of/together with the ‘site characteristics’ would compromise results since there would be a tendency to group together all sites that have high outliers, even though these outliers result from random fluctuations, and testing for the homogeneity of the formed regions by a statistic calculated from the ‘at-site statistics’ would be misleading (Smithers and Schulze, 2001).

The Ward’s method (which tends to form clusters with equal number of sites) were applied as clustering algorithms (Guttmann, 1993). Reasonable numbers of clusters are 4 and 3 for Ward’s method, homogeneity tests for all sites taken as one region was performed as well as in table 3. However, subjective adjustments (mainly according to the site location and it’s climatologically characteristics) are necessary in all cases to improve the geographical and climatologically coherence of regions and to avoid heterogeneity.

C. Testing for homogeneity of regions

Tests for the homogeneity of regions are usually based on a quantity that measures some aspect of the frequency distribution, e.g. the combination of L-CV, \( t_3 \) and the L-kurtosis \( t_4 \) (Hosking and Wallis, 1993; Adamowski, 2000), and compare the ‘at-site’ estimates with the regional estimate of this quantity. (See Appendix I for a formal definition of L-
CV, τ3 and τ4). The mean and standard deviation of the chosen dispersion measure are obtained by a simulation of a homogeneous region with sites having record lengths the same as the observed data (Monte Carlo method). The test employed in the present study is that of Hosking and Wallis (1993); (see Appendix II for their description).

The heterogeneity measure compares the between site variations in sample L-moments for the group of sites with what would be expected for a homogenous region. A visual assessment of the dispersion of the at-site values can be obtained by plotting them on a graph of L-skewness versus L-CV or L-kurtosis as in figure 2.

To identify whether the between-site dispersion of the sample L-moments for the group of sites is larger than what would be expected for a homogeneous region, and to establish what is to be expected, 500 repeated simulations of a homogeneous region is obtained.

As a result of the regional (17 sites) analysis, values of 1.56, -0.96, and -1.59 were obtained for the three H measures, respectively. The region is considered to be possibly homogeneous. As it has presented in table 3.

In this study, the accuracy of the estimates for selected region(s) is assessed, using a Monte Carlo simulation procedure. Simulated sites are assumed to have the same record lengths as those in sample data. The number of repetitions, M, is set to 10000, and the number of simulations is set to 500.

Data generated for each site are then fitted to the sample regional distribution, and the simulated dimensionless quantile estimates for each site and the region are computed (as in table 4). Then, it is possible to obtain the flood estimates for each site by multiplying the dimensionless quantiles with the sample means for each site.

Using the simulation program, bias and root mean square errors (RMSE) of the quantile estimates for each site and the accuracies of the estimated quantiles are determined through relative BIAS, relative RMSE, and regional average absolute relative BIAS, respectively. These results compare the simulated and observed estimates, and the accuracy of the estimated quantiles are obtained. Generally, for each simulated region, 90% error bounds for the regional growth curve are computed, based on the selected nonexceedance probabilities, as shown in figure 3. The 90% error limits of the lower (0.95PT) and the upper (0.05PT) are calculated and listed in table 5. Furthermore, simulation results of BIAS and RMSE values for the three regions are listed in table 6.

VII. CONCLUSION

Flood frequency modeling is one of the simplest and widely used applications of statistics in the field of hydrology. The L-moments have been used for parameter estimation, homogeneity testing and selection of the regional distribution in the study.

As it shown in table (3) the person type three (PE3) value has the best goodness-of-fit with the data at the region, and three parameter general logistic, general extreme value, general normal and general Pareto are the acceptable distributions. The Z value of these distributions is less than 1.64, and the general Person type III has the lowest value of Z statistic.

The results in table 5 confirm that quantile estimates generally become less accurate at larger return periods. On the other hand, RMSE values in the quantile estimates for the homogenous region HR are less for the larger return period.

Table 6 confirms that RMSE values of the estimated quantiles are always greater than the RMSE values of the growth curve, corresponding to the same return periods. This is due to the fact that RMSE’s of the growth curve contain a contribution from the variability of the estimated growth curve only, while RMSE’s of the quantiles contain a further contribution from the variability of the estimated index flood (single site sample mean).

For the homogenous region HR, since the expected bias at each site are equal, its regional average absolute relative bias and the regional average relative bias of the estimated quantile are equal. Thus, in practice, it is difficult to ensure that the sites used in a particular application of regional frequency analysis constitute a region that is exactly homogenous. Accordingly, the utility of regional frequency analysis depends on whether its performance is acceptable for moderately heterogeneous regions (Hosking & Wallis, 1997).

The L-moment ratio diagram is a graph between L-kurtosis and L-skewness and can be used to select a suitable probability distribution for a region. Theoretical curves of various distributions as well as the regional L-skewness and L-kurtosis are plotted on same graph to select the best-fit distribution for the basin as has been represented in figure 4. As the same as the graphical representation of the regional growth curve for the data has been presented in figure 5.
Fig. 1 Location of Luan River Basin in China and sites of the study area.

Fig. 2 Diagram of sample L-moments ratios.

Fig. 4 The L-moments ratio diagram for the basin.

Fig. 5 The Regional Growth Curve.

Fig. 3 Regional growth curve and bounds.
### TABLE I. (A) HYDROLOGICAL AND PHYSICAL CHARACTERISTICS FOR THE STUDY AREA

<table>
<thead>
<tr>
<th>River name</th>
<th>Stat. name</th>
<th>Cod Of stations</th>
<th>Records years</th>
<th>l1</th>
<th>t</th>
<th>t3</th>
<th>t4</th>
<th>t5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daluanhe</td>
<td>Waigou menzi zhan</td>
<td>2</td>
<td>15</td>
<td>0.0949</td>
<td>0.2697</td>
<td>0.291</td>
<td>0.0529</td>
<td>-0.1085</td>
</tr>
<tr>
<td>Luanhe</td>
<td>jiutunzhan</td>
<td>4</td>
<td>7</td>
<td>0.3697</td>
<td>0.4757</td>
<td>0.2145</td>
<td>-0.2335</td>
<td>-0.367</td>
</tr>
<tr>
<td>Luanhe</td>
<td>sandaohezizhan</td>
<td>6</td>
<td>17</td>
<td>0.5207</td>
<td>0.4917</td>
<td>0.5277</td>
<td>0.3302</td>
<td>0.2127</td>
</tr>
<tr>
<td>Luanhe</td>
<td>panjiakouzhan</td>
<td>11</td>
<td>20</td>
<td>3.5035</td>
<td>0.577</td>
<td>0.4888</td>
<td>0.2889</td>
<td>0.2072</td>
</tr>
<tr>
<td>Luanhe</td>
<td>Luojiatun zhan</td>
<td>12</td>
<td>21</td>
<td>4.8957</td>
<td>0.57</td>
<td>0.5275</td>
<td>0.3764</td>
<td>0.2586</td>
</tr>
<tr>
<td>Luanhe</td>
<td>Luanxian zhan</td>
<td>13</td>
<td>33</td>
<td>7.1233</td>
<td>0.5556</td>
<td>0.5159</td>
<td>0.3187</td>
<td>0.1986</td>
</tr>
<tr>
<td>Xiaoluanhe</td>
<td>Goutaizi zhan</td>
<td>21</td>
<td>13</td>
<td>0.0628</td>
<td>0.3512</td>
<td>0.4133</td>
<td>0.2158</td>
<td>0.0282</td>
</tr>
<tr>
<td>Yixunhe</td>
<td>Weichang zhan</td>
<td>26</td>
<td>12</td>
<td>0.0536</td>
<td>0.4524</td>
<td>0.5347</td>
<td>0.3996</td>
<td>0.3589</td>
</tr>
<tr>
<td>Yixunhe</td>
<td>Miaogong shiku</td>
<td>27</td>
<td>11</td>
<td>0.079</td>
<td>0.3522</td>
<td>0.2501</td>
<td>0.358</td>
<td>0.4706</td>
</tr>
<tr>
<td>Yimatuhe</td>
<td>Xiahenan zhan</td>
<td>33</td>
<td>13</td>
<td>0.12</td>
<td>0.5261</td>
<td>0.5541</td>
<td>0.4126</td>
<td>0.2614</td>
</tr>
<tr>
<td>Wuiehe</td>
<td>Chengde zhan</td>
<td>34</td>
<td>16</td>
<td>0.4448</td>
<td>0.6822</td>
<td>0.5883</td>
<td>0.3116</td>
<td>0.0663</td>
</tr>
<tr>
<td>Xinglonghe</td>
<td>xiaoxishanzhan</td>
<td>35</td>
<td>11</td>
<td>0.0451</td>
<td>0.5822</td>
<td>0.4232</td>
<td>0.047</td>
<td>-0.1598</td>
</tr>
<tr>
<td>Luihe</td>
<td>liyingzhan</td>
<td>37</td>
<td>16</td>
<td>0.3514</td>
<td>0.58</td>
<td>0.4364</td>
<td>0.1655</td>
<td>0.0521</td>
</tr>
<tr>
<td>Puhe</td>
<td>Pingquan zhan</td>
<td>38</td>
<td>9</td>
<td>0.0875</td>
<td>0.6769</td>
<td>0.6906</td>
<td>0.6139</td>
<td>0.5831</td>
</tr>
<tr>
<td>Qinglonghe</td>
<td>Taolinkou zhan</td>
<td>43</td>
<td>15</td>
<td>2.052</td>
<td>0.7016</td>
<td>0.6434</td>
<td>0.3666</td>
<td>0.2006</td>
</tr>
<tr>
<td>Shihe</td>
<td>Xiaochen zhuang zhan</td>
<td>49</td>
<td>15</td>
<td>0.4455</td>
<td>0.554</td>
<td>0.4559</td>
<td>0.1831</td>
<td>0.0026</td>
</tr>
<tr>
<td>Douhe</td>
<td>Douhe shuiku</td>
<td>68</td>
<td>18</td>
<td>0.1113</td>
<td>0.5849</td>
<td>0.5331</td>
<td>0.3163</td>
<td>0.1873</td>
</tr>
</tbody>
</table>

### TABLE I. (B) HYDROLOGICAL AND PHYSICAL CHARACTERISTICS FOR THE STUDY AREA

<table>
<thead>
<tr>
<th>River name</th>
<th>Stat. name</th>
<th>Cod Of stations</th>
<th>Records years</th>
<th>Area Km²</th>
<th>Lat. (ºN)</th>
<th>Long. (ºE)</th>
<th>ELEV (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daluanhe</td>
<td>Waigou menzi zhan</td>
<td>2</td>
<td>15</td>
<td>8930</td>
<td>41.9166</td>
<td>116.6166</td>
<td>1165.5</td>
</tr>
<tr>
<td>Luanhe</td>
<td>jiutunzhan</td>
<td>4</td>
<td>7</td>
<td>1890</td>
<td>41.65</td>
<td>117.05</td>
<td>1091.1</td>
</tr>
<tr>
<td>Luanhe</td>
<td>sandaohezizhan</td>
<td>6</td>
<td>17</td>
<td>14000</td>
<td>41.35</td>
<td>117.35</td>
<td>1026</td>
</tr>
<tr>
<td>Luanhe</td>
<td>panjiakouzhan</td>
<td>11</td>
<td>20</td>
<td>14280</td>
<td>40.9667</td>
<td>117.7</td>
<td>769.5</td>
</tr>
<tr>
<td>Luanhe</td>
<td>Luojiatun zhan</td>
<td>12</td>
<td>21</td>
<td>2500</td>
<td>41.95</td>
<td>117.7667</td>
<td>1083.9</td>
</tr>
<tr>
<td>Luanhe</td>
<td>Luanxian zhan</td>
<td>13</td>
<td>33</td>
<td>2500</td>
<td>41.7167</td>
<td>117.8333</td>
<td>995.1</td>
</tr>
<tr>
<td>Site</td>
<td>Location</td>
<td>Code</td>
<td>Station</td>
<td>X</td>
<td>Y</td>
<td>L</td>
<td>N</td>
</tr>
<tr>
<td>--------------</td>
<td>----------------------</td>
<td>------</td>
<td>---------</td>
<td>---</td>
<td>---</td>
<td>----</td>
<td>-----</td>
</tr>
<tr>
<td>Xiaoluanhe</td>
<td>Goutaizi zhan</td>
<td>21</td>
<td>13</td>
<td>2640</td>
<td>41.3</td>
<td>117.7</td>
<td>922.8</td>
</tr>
<tr>
<td>Yixunhe</td>
<td>Weichang zhan</td>
<td>26</td>
<td>12</td>
<td>200</td>
<td>41.1833</td>
<td>117.9333</td>
<td>225.6</td>
</tr>
<tr>
<td>Yixunhe</td>
<td>Miaogong shuiku</td>
<td>27</td>
<td>11</td>
<td>2200</td>
<td>40.9667</td>
<td>117.9333</td>
<td>589.7</td>
</tr>
<tr>
<td>Yimatuhe</td>
<td>Xiahezhan</td>
<td>33</td>
<td>13</td>
<td>193</td>
<td>41.1833</td>
<td>117.9333</td>
<td>436.5</td>
</tr>
<tr>
<td>Wuliehe</td>
<td>Chengde zhan</td>
<td>34</td>
<td>16</td>
<td>446</td>
<td>40.9833</td>
<td>118.7</td>
<td>707.4</td>
</tr>
<tr>
<td>Xinglonghe</td>
<td>Xiaoxishan zhan</td>
<td>35</td>
<td>11</td>
<td>33700</td>
<td>40.4167</td>
<td>118.3</td>
<td>721.6</td>
</tr>
<tr>
<td>Liuhe</td>
<td>Liyingzhan</td>
<td>37</td>
<td>16</td>
<td>37100</td>
<td>40.15</td>
<td>118.65</td>
<td>107.4</td>
</tr>
<tr>
<td>Puhe</td>
<td>Pingquan zhan</td>
<td>38</td>
<td>9</td>
<td>5250</td>
<td>40.1333</td>
<td>119.5</td>
<td>114.6</td>
</tr>
<tr>
<td>Qinglonghe</td>
<td>Taolinkou zhan</td>
<td>43</td>
<td>15</td>
<td>44100</td>
<td>39.7333</td>
<td>118.75</td>
<td>32.7</td>
</tr>
<tr>
<td>Shiheng</td>
<td>Xiaochen zhuang zhan</td>
<td>49</td>
<td>15</td>
<td>220</td>
<td>40.0333</td>
<td>119.7167</td>
<td>65.7</td>
</tr>
<tr>
<td>Douhe</td>
<td>Douhe shuiku</td>
<td>68</td>
<td>18</td>
<td>647</td>
<td>39.7333</td>
<td>118.3</td>
<td>72.9</td>
</tr>
</tbody>
</table>

**TABLE II**

<table>
<thead>
<tr>
<th>Site</th>
<th>Parameter Values for Each Site</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X1</td>
</tr>
<tr>
<td>1</td>
<td>0.8632</td>
</tr>
<tr>
<td>2</td>
<td>0.8189</td>
</tr>
<tr>
<td>3</td>
<td>0.5898</td>
</tr>
<tr>
<td>4</td>
<td>0.5449</td>
</tr>
<tr>
<td>5</td>
<td>0.5246</td>
</tr>
<tr>
<td>6</td>
<td>0.5439</td>
</tr>
<tr>
<td>7</td>
<td>0.7575</td>
</tr>
<tr>
<td>8</td>
<td>0.619</td>
</tr>
<tr>
<td>9</td>
<td>0.8449</td>
</tr>
<tr>
<td>10</td>
<td>0.546</td>
</tr>
<tr>
<td>11</td>
<td>0.3879</td>
</tr>
<tr>
<td>12</td>
<td>0.5901</td>
</tr>
<tr>
<td>13</td>
<td>0.5813</td>
</tr>
<tr>
<td>14</td>
<td>0.3372</td>
</tr>
<tr>
<td>15</td>
<td>0.3367</td>
</tr>
<tr>
<td>16</td>
<td>0.5859</td>
</tr>
<tr>
<td>17</td>
<td>0.5085</td>
</tr>
</tbody>
</table>

**TABLE III**

<table>
<thead>
<tr>
<th>Region</th>
<th>Heterogeneity measure</th>
<th>Goodness of fit</th>
<th>Distribution function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H1</td>
<td>H2</td>
<td>H3</td>
</tr>
<tr>
<td>HR</td>
<td>1.56</td>
<td>-0.96</td>
<td>-1.59</td>
</tr>
</tbody>
</table>
**TABLE IV**
THE SIMULATED DIMENSIONLESS QUANTILE ESTIMATES FOR EACH SITE IN THE BASIN

<table>
<thead>
<tr>
<th>SITE</th>
<th>0.01</th>
<th>0.1</th>
<th>0.5</th>
<th>0.9</th>
<th>0.99</th>
<th>0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.353</td>
<td>0.499</td>
<td>0.863</td>
<td>1.658</td>
<td>2.961</td>
<td>4.599</td>
</tr>
<tr>
<td>2</td>
<td>-0.308</td>
<td>0.06</td>
<td>0.819</td>
<td>2.159</td>
<td>3.983</td>
<td>5.966</td>
</tr>
<tr>
<td>3</td>
<td>0.194</td>
<td>0.261</td>
<td>0.59</td>
<td>2.049</td>
<td>6.516</td>
<td>15.605</td>
</tr>
<tr>
<td>4</td>
<td>-0.002</td>
<td>0.1</td>
<td>0.545</td>
<td>2.284</td>
<td>7.053</td>
<td>15.971</td>
</tr>
<tr>
<td>5</td>
<td>0.065</td>
<td>0.143</td>
<td>0.525</td>
<td>2.217</td>
<td>7.392</td>
<td>17.919</td>
</tr>
<tr>
<td>6</td>
<td>0.073</td>
<td>0.156</td>
<td>0.544</td>
<td>2.202</td>
<td>7.109</td>
<td>16.838</td>
</tr>
<tr>
<td>7</td>
<td>0.314</td>
<td>0.413</td>
<td>0.758</td>
<td>1.826</td>
<td>4.216</td>
<td>8.037</td>
</tr>
<tr>
<td>8</td>
<td>0.266</td>
<td>0.325</td>
<td>0.619</td>
<td>1.957</td>
<td>6.135</td>
<td>14.771</td>
</tr>
<tr>
<td>9</td>
<td>0.092</td>
<td>0.323</td>
<td>0.845</td>
<td>1.861</td>
<td>3.367</td>
<td>5.118</td>
</tr>
<tr>
<td>10</td>
<td>0.169</td>
<td>0.229</td>
<td>0.564</td>
<td>2.084</td>
<td>7.188</td>
<td>18.143</td>
</tr>
<tr>
<td>11</td>
<td>-0.028</td>
<td>0.032</td>
<td>0.388</td>
<td>2.332</td>
<td>9.45</td>
<td>26.085</td>
</tr>
<tr>
<td>12</td>
<td>-0.119</td>
<td>0.036</td>
<td>0.59</td>
<td>2.361</td>
<td>6.429</td>
<td>13.064</td>
</tr>
<tr>
<td>13</td>
<td>-0.092</td>
<td>0.051</td>
<td>0.581</td>
<td>2.345</td>
<td>6.54</td>
<td>13.57</td>
</tr>
<tr>
<td>14</td>
<td>0.104</td>
<td>0.128</td>
<td>0.337</td>
<td>2.026</td>
<td>10.691</td>
<td>36.928</td>
</tr>
<tr>
<td>15</td>
<td>0.017</td>
<td>0.055</td>
<td>0.337</td>
<td>2.22</td>
<td>10.438</td>
<td>32.407</td>
</tr>
<tr>
<td>16</td>
<td>-0.012</td>
<td>0.109</td>
<td>0.586</td>
<td>2.268</td>
<td>6.482</td>
<td>13.834</td>
</tr>
<tr>
<td>17</td>
<td>0.048</td>
<td>0.125</td>
<td>0.509</td>
<td>2.24</td>
<td>7.621</td>
<td>18.705</td>
</tr>
</tbody>
</table>

**TABLE V**
SIMULATION RESULTS FOR ESTIMATED REGIONAL QUANTILES, THEIR CORRESPONDING ERROR BOUNDS AND RMSE VALUES

<table>
<thead>
<tr>
<th>F</th>
<th>qhat(F)</th>
<th>RMSE</th>
<th>Error bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.569</td>
<td>3.508</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.753</td>
<td>1.186</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.994</td>
<td>0.321</td>
<td>0.724</td>
</tr>
<tr>
<td>0.9</td>
<td>1.254</td>
<td>0.091</td>
<td>1.153</td>
</tr>
<tr>
<td>0.99</td>
<td>1.48</td>
<td>0.382</td>
<td>1.068</td>
</tr>
<tr>
<td>0.999</td>
<td>1.654</td>
<td>0.675</td>
<td>0.913</td>
</tr>
</tbody>
</table>

**TABLE VI**
BIAS, ABSOLUTE BIAS, AND RMSE OF THE ESTIMATE QUANTILES AND THE CORRESPONDING QUANTITIES WITHIN THE ESTIMATED GROWTH CURVE

<table>
<thead>
<tr>
<th>Region</th>
<th>Average Quantiles</th>
<th>Growth curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>F = :</td>
<td>0.9000</td>
<td>0.9900</td>
</tr>
<tr>
<td>HR</td>
<td>ABS. bias</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>Bias</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.453</td>
</tr>
</tbody>
</table>

**APPENDIX**

I. L-MOMENTS

**A. An Introduction**

Hosking (1990) stressed that conventional moments are not always satisfactory in two major aspects. They do not always import easily interpreted information about the shape of a distribution, and parameter estimates of distributions fitted by the moments are often less accurate than those obtained by other methods, such as the maximum likelihood. L-moments are an alternative to conventional moments and can be estimated by linear combinations of order statistics. L-moments have the theoretical advantages over conventional moments of being able to characterize a wider range of distributions and, when estimated from a sample, of being more robust to the presence of outliers in the data. Experience also shows that, compared with conventional moments, L-moments are less subject to bias in estimation and approximate their asymptotic normal distribution more closely in finite samples (Hosking, 1990).
L-moments approach covers the characterization of probability distributions, the summary of observed data samples, the fitting of probability distributions to data, and testing the hypothesis about the distributional form. The L in L-moments emphasizes the linearity. The mean, variance, and skewness are defined in terms of moments as: L-mean, L-scale and L-skewness, respectively.

The first L-moment is identical to the usual mean and is a measure of location. The second L-moment is a measure of the spread or dispersion of the data. The third L-moment is a measure of the symmetry of the data. Suppose there are three sample points: \( x_1 < x_2 < x_3 \). If \( x_1 \) and \( x_3 \) are symmetrical about the central point, then \( x_3 - x_2 = x_2 - x_1 \) and, thus, \( x_3 - 2x_2 + x_1 = 0 \). If \( x_3 \) is further away from \( x_2 \) than \( x_1 \), then the distribution has positive skewness and \( x_3 - 2x_2 + x_1 > 0 \). Similarly for negative skewness, this value will be smaller than zero. The linear combination, \( x_3 - 2x_2 + x_1 \), is called the second-order difference of the ordered sample, and the third L-moment is determined from the average of the linear combination of this type. The fourth L-moment can be thought of as a measure of the peakiness of the data, and it is based on the third-order difference of the ordered sample. Thus, the L-moments of a random variable \( X \) are formally defined as follows:

\[
\begin{align*}
\lambda_1 &= E(X) \\
\lambda_2 &= \frac{E(X^2) - E(X)}{\lambda_1^2} \\
\lambda_3 &= \frac{E(X^3) - 3E(X)E(X^2) + 2(E(X))^3}{\lambda_1^3} \\
\lambda_4 &= \frac{E(X^4) - 4E(X)E(X^3) + 6(E(X))^2E(X^2) - 3(E(X))^4}{\lambda_1^4}
\end{align*}
\]

where \( \lambda_1, \lambda_2, \lambda_3, \ldots \) are the theoretical L-moments and \( X_{i:n} \) denotes the \( i \)th observation from an ordered sample of size \( n \). Thus \( E[X_{2:2} - X_{1:2}] \) is the expected value of the difference between the largest and the second largest observations in a sample of size two. \( \lambda_r \) is used to denote a theoretical L-moment of a distribution, and \( \lambda_r \) denotes the sample estimates of the L-moments, where each takes the units of the original data.

### B. Index Flood Procedure Based on L-moments

Assume \( N \) sites in a region, each with a sample size \( n_i \) at site \( i \); then, let \( Q_i(F) \) be the quantile of nonexceedance probability \( F \) at site \( i \). The index flood procedure assumes that the region is homogeneous; that is, the frequency of the \( N \) sites is identical apart from a site-specific scaling factor. The index flood is identified as:

\[
Q_i(F) = u_i q(F) \quad i = 1, \ldots, N
\]

where \( u_i \) is the index flood. In this study, \( u_i \) is supposed to be the mean of the at-site frequency distribution, and \( q(F) \) is the regional quantile of \( F \).

The regional quantiles \( q(F) \), of \( 0 < F < 1 \), form the regional growth curve, which defines a dimensionless regional frequency distribution common to all sites. The sample mean at site \( i \) is estimated by \( \hat{u}_i = n^{-1} \sum_{j=1}^{n_i} Q_i(j) \), and the dimensionless rescaled data by \( q_{ij} = Q_{ij} / \hat{u}_i \), \( j = 1, 2, \ldots, n_i, i = 1, \ldots, N \). It is usually assumed that the form of \( q(F) \) is known, apart from \( p \) undetermined parameters \( \theta_1, \ldots, \theta_p \). Hosking & Wallis (1993) considered an index flood procedure in which the parameters are estimated separately at each site. They denoted the site \( i \) estimate of \( \theta_k \) by \( \hat{\theta}_k^i \) and then combined the at-site estimates to give regional estimates:

\[
\sum_{i=1}^{N} n_i \hat{\theta}_k^i / \sum_{i=1}^{N} n_i = \theta_k^R
\]

Substitution of these estimates into \( q(F) \) gives the estimated regional quantile:

\[
q(F) = q(F; \hat{\theta}_1^R, \ldots, \hat{\theta}_p^R)
\]

(1993). The at-site \( i \) quantile estimates are obtained by combining the estimates of \( u_i \) and \( q(F) \): \( \hat{Q}_i(F) = \hat{u}_i \hat{q}(F) \).

### C. L-moments and PWMs

Parameter estimation methods commonly in use include conventional moments, maximum likelihood, probability weighted moments, and L-moments in the form of PWMs (probability weighted moments). L-moments are an alternative way to summarize the statistical properties of the hydrologic data and to describe the shape of the probability distributions. L-moments arose as modifications of the PWMs, and the main point behind the theory of L-moments is the order statistics, the kind of information that can be obtained from the ordered observations, and the relations between these observations and the moments (location, scale, and shape parameters).

The theory of L-moments parallels the theory of conventional moments. Its main advantage over conventional moments is that L-moments, being linear functions of the data, suffer less from the effects of sampling variability. L-moments are more robust than the conventional moments to outliers in the data and enable more secure inferences to be made from small samples about an underlying probability distribution. They sometimes yield more efficient parameter estimates than the maximum likelihood estimates. Hosking (1990) demonstrated that L-moment methods perform competitively with the best available statistical techniques.

### D. L-moment Estimator

For a random variable \( X \) with the cumulative distribution function \( F \), L-moments can be written as functions of PWMs. Greenwood et al. (1979) expressed the PWMs as:
where $p$, $r$ and $s$ are nonnegative integers. The above equation can be formulated into two different population PWMs; they are:

$$
\alpha_r = M_{0,r,s} = E \left\{ X \left[ 1 - F(X) \right]^r \right\} 
$$

$$
\beta_r = M_{1,r,s} = E \left\{ X F(X) \right\}^r 
$$

PWMs are the expectation of $X$ times powers of $F(X)$. Estimators of L-moments are simply written as linear functions of estimators of PWMs. The first PWM estimator $b_0$ of $\beta_0$ is the sample mean. To estimate the other PWMs, one employs the ordered observations, or the order statistics $X(n) \leq \ldots \leq X(1)$, corresponding to the ranked observations in a sample $X_i$, $i=1, \ldots, n$. A simple estimator of $\beta_r$ for $r \geq 1$ is:

$$
b^*_r = n^{-1} \sum_{j=1}^{n} X_j \left[ 1 - \left( j - 0.35 \right) / n \right]^r
$$

where $(1-(j-0.35)/n)$ are estimators of $F(X_j)$. $b^*_r$ is suggested for use when estimating quantiles and fitting a distribution at a single site. Although it is biased, it generally yields smaller mean square error quantile estimators than the unbiased estimators (Stedinger et al., 1993). The L-moment calculation proceeds via estimation of PWMs (Greenwood et al., 1979). The unbiased PWM estimators are:

$$
b_0 = \frac{1}{n} \sum_{i=1}^{n} X_i 
$$

$$
b_1 = \frac{1}{n-1} \sum_{j=1}^{n} (j-1) X_j / (n-1) 
$$

$$
b_2 = \frac{1}{n-2} \sum_{j=1}^{n} (j-1)(j-2) X_j / [(n-1)(n-2)] 
$$

$$
b_3 = \frac{1}{n-3} \sum_{j=1}^{n} (j-1)(j-2)(j-3) X_j / [(n-1)(n-2)(n-3)] 
$$

and in the general formula are:

$$
\hat{\beta}_r = b_0 \frac{1}{n} \sum_{j=1}^{n} \left[ (j-1)(j-2)(\ldots)(j-r) X_j / [(n-1)(n-2)(\ldots)(n-r)] \right] \text{ for } r \geq 1. 
$$

where $n$ is the sample size and $X_j$ denotes the $j$th element of a sample of size $n$ sorted into ascending order. Unbiased estimators are recommended for deriving L-moment diagrams and for use with regionalization procedures where unbiasedness is important. Then, L-moments can be easily calculated in terms of PWMs for any distribution. The first four sample L-moments are then estimated by:

$$
l_1 = b_0 
$$

$$
l_2 = 2b_1 - b_0 
$$

$$
l_3 = 6b_2 - 6b_1 + b_0 
$$

$$
l_4 = 20b_3 - 30b_2 + 12b_1 - b_0 
$$

(12)

and in their theoretical forms as:

$$
\lambda_1 = E(X) = \beta_0 
$$

$$
\lambda_2 = [E(X_2) - X_1^2] / 2 = 2b_1 - b_0 
$$

$$
\lambda_3 = [E(X_3) - 3X_2X_1 + X_1^3] / 3 = 6b_2 - 6b_1 + b_0 
$$

$$
\lambda_4 = [E(X_4) - 4X_3X_2 + 6X_2^2X_1 - 4X_1X_2^2 + X_1^4] / 4 = 20b_3 - 30b_2 + 12b_1 - b_0 
$$

(13)

where estimates of the $\lambda_i$ are obtained by replacing the unknown $\beta_r$ by sample estimators $b_r$. L-moment ratios are the quantiles (Hosking & Wallis, 1993). The use of L-moments to describe probability distributions is justified and proved by Hosking (1990). He has demonstrated that a distribution may be specified by its L-moments even if some of its conventional moments do not exist and that such a specification is always unique. The measure of the scale $\lambda_2$ of the random variable $X$ is often convenient to standardize the higher moments $\lambda_1$, $\lambda_2$, $\lambda_3$, so that they are independent of the units of measurement of $X$. Therefore, the L-moment ratios are used as estimation procedures for obtaining growth curves and are defined by:

$$
\tau_1 = \lambda_1 / \lambda_2 
$$

$$
\tau_2 = \lambda_2 / \lambda_2 
$$

$$
\tau_3 = \lambda_3 / \lambda_2 
$$

$$
\tau_4 = \lambda_4 / \lambda_2 
$$

(13)

In particular, $\lambda_1$ is the mean of the distribution or measure of location; $\lambda_2$ is a measure of scale; $\lambda_3$ is a measure of skewness, and $\lambda_4$ is a measure of kurtosis. The L-CV, $\tau = \lambda_2 / \lambda_1$, is analogous to the usual coefficient of variation. L-skewness and L-kurtosis are both defined relative to the L-scale, $\lambda_2$, and sample estimates of L-moment ratios can be written as $t_1$, $t_3$, and $t_4$.

APPENDIX II

II. STEPS OF APPLICATION

A. Diagnostic Statistical Tests

Hosking & Wallis (1993) addressed that regional frequency analysis involves four stages, the first three of which involve
subjective judgment: (a) screening of data by means of the discordance measure, $D_i$, which provides an initial screening of the data and identifies unusual sites in a region; (b) identification of homogeneous regions which is the assignment of the sites to regions by the means of heterogeneity measure, H, that performs the test by calculating summary statistics (sample L-moments) of the at-site data and compares the between-site variability of these statistics with what would be expected of a homogeneous region; (c) choice of a regional frequency distribution by means of the goodness-of-fit measure, $Z$, which assesses whether a candidate distribution provides an adequate fit to the data; and (d) the estimation of regional frequency distribution.

B. Discordancy Measure, $D_i$, Test for Side Data

When a single site does not appear to belong to the cloud of $(\tau_3, \tau_4)$ points on the L-moment diagram, a test of discordance can be used to determine whether it should be removed from the region. Discordance measure, $D_i$, is used to screen out the data from unusual sites; and the test is applied by calculating the D-statistic, which is defined in terms of L-moments. Let $U_i = [\tau_{i3}, \tau_{i4}]^T$ be a vector containing the L-moments. If the group averages $\overline{U}$ and sample covariance matrix $S$ are defined as:

$$ \overline{U} = \frac{1}{n} \sum_{i=1}^{n} U_i $$

$$ S = \frac{1}{(N-1)} \sum_{i=1}^{n} (U_i - \overline{U})(U_i - \overline{U})^T $$

then the discordancy measure for site $i$ is:

$$ D_i = 3^{-1} (U_i - \overline{U})^T S^{-1} (U_i - \overline{U}) $$

Where $n$ is the total number of sites. Large values of $D_i$ indicate sites that are the most discordant from the group as a whole and are most worthy of investigation for the presence of data errors (Hosking & Wallis, 1993).

The average of $D_i$ over all sites is 1, and it is not easy to choose a single value of $D_i$ that can be used as a criterion for deciding whether a site is unusual. Based on an assumption that the $U_i$ are drawn from independent identical multivariate normal distributions, the statistics $3D_i$ have approximately independent chi-square distributions with three degrees of freedom for large regions. Furthermore, approximately 3% of the $D_i$ values exceed 3 (Hosking & Wallis, 1993). If a site’s D-statistic exceeds 3, its data are considered to be discordant from the rest of the regional data. Two possibilities can be investigated: either there may be an error in the data or the station may properly belong to another region or no region at all. However, it is advisable to examine the data for the sites with the largest $D_i$ values, regardless of the magnitude of these values (Hosking & Wallis, 1993). Hosking & Wallis (1997) considered that $D_i$ largely depends on the number of sites in the group, and they gave critical values for it. Thus, initially, Hosking & Wallis (1993) suggested the criterion $D_i \geq 3$, but this is not satisfactory for small regions. In their recent studies (Hosking & Wallis, 1997), they have defined the following algebraic bound for the critical $D_i$ value: $D_i < (N - 1)/3$, and the value of $D_i > 3$ can only occur in regions having 11 or more sites.

C. Heterogeneity Test

L-moment heterogeneity tests assess whether a group of sites might reasonably be treated as a homogenous region. The heterogeneity measure compares the between-site variations in sample L-moments for the group of sites with what would be expected for a homogeneous region (Hosking & Wallis, 1993).

The homogeneity test used in this study is the test that is proposed by Hosking & Wallis (1993) and is based on various orders of sample L-moment ratios. It is particularly based on the variability of three different levels of tests: a test based on the L-CV only; a test based on the L-CV and L-skewness; and a test based on the L-skewness and L-kurtosis. These tests are called the V-statistic and are respectively defined as:

$$ V_1 = \frac{\sum_{i=1}^{n} \mu_{v1}}{\sum_{i=1}^{n} \sigma_{v1}} $$

$$ V_2 = \frac{\sum_{i=1}^{n} \mu_{v2}}{\sum_{i=1}^{n} \sigma_{v2}} $$

$$ V_3 = \frac{\sum_{i=1}^{n} \mu_{v3}}{\sum_{i=1}^{n} \sigma_{v3}} $$

The heterogeneity measure is then defined as:

$$ H_k = (\mu_{v k} - \mu v) / \sigma v k; \quad k = 1, 2, 3 $$

where $\mu v k$ and $\sigma v k$ are the mean and the standard deviation of the simulated values of the V, respectively. $V_k$ is calculated from the regional data and is based on corresponding V-statistics: $V_1$, $V_2$, and $V_3$. Then, a region can be declared homogeneous with a corresponding order of L-moment if $H < 1$; the region is possibly homogeneous if $1 \leq H < 2$; and it is definitely declared heterogeneous if $H \geq 2$.

Hosking & Wallis (1993) mentioned that, in assessing the heterogeneity measures of a proposed region, $H_k$ values are to be compared with their corresponding given criteria. If the region is not acceptably homogenous, some redefinition of the region should be considered. The region could be divided into two or more sub regions, some sites could be removed from the region, or a completely different assignment of sites to regions could be tried.

Hosking & Wallis (1993) assessed the performance of $H_1$ as a heterogeneity measure in a series of Monte Carlo simulation experiments. To achieve reliable estimates $\mu v$ and
is the average L-kurtosis value computed from observed data. For a selected distribution, Z is defined by Hosking & Wallis (1993) as:

$$Z_{\text{dist}} = \left( \tau_4^{\text{dist}} - \tau_4 + \beta_4 \right) / \sigma_4$$ \hspace{1cm} (19)

where $\tau_4$ is the average L-kurtosis value computed from the data of a given region; $\tau_4^{\text{dist}}$ is the average L-kurtosis value computed from simulation for a fitted distribution; $\beta_4$ is the bias of $\tau_4$; and $\sigma_4$ is the standard deviation of L-kurtosis values obtained from simulation. Thus:

$$\beta_4 = \left( \frac{1}{N_{\text{sim}}} \sum_{m=1}^{N_{\text{sim}}} \left( \bar{\tau}_4^m - \tau_4 \right) \right)^{-1}$$ \hspace{1cm} (20)

$$\sigma_4 = \left[ \left( \frac{1}{N_{\text{sim}}} \sum_{m=1}^{N_{\text{sim}}} \left( \bar{\tau}_4^m - \tau_4 \right)^2 - \left( \frac{1}{N_{\text{sim}}} \sum_{m=1}^{N_{\text{sim}}} \bar{\tau}_4^m \beta_4 \right)^2 \right)^{-1} \right]^{1/2}$$ \hspace{1cm} (21)

where $\bar{\tau}_4^m$ is the regional average L-kurtosis and is to be calculated for the mth simulated region. Then, a given distribution is declared to be of adequate fit if $Z_{\text{dist}}$ is sufficiently close to zero. A reasonable criterion is $|Z_{\text{dist}}| \leq 1.64$. This criterion corresponds to acceptance of the hypothesized distribution at a confidence level of 90% and shows approximately a standard normal distribution if the at-site L-kurtosis statistics have independent identical normal distributions.

For a region that is acceptably homogenous, the calculated Z values flag as acceptable for all distributions which satisfy the Z criteria; and regional growth curves for all accepted distributions are calculated. If these curves are all approximately equal, then any of the acceptable distributions is considered adequate. If the curves are not approximately equal, there is a problem of scarcity of data. In this case, it may be better to use the five parameter Wakeby distribution, which is particularly robust to misspecification of the underlying distribution function of a homogenous region. It may happen that none of the candidate distributions is accepted by the Z criteria, this may occur when the number of sites in the region or the at-site record lengths are large. In these circumstances, $\sigma_4$ is small, and Z can be large even if the regional average L-skewness and L-kurtosis are fairly close to those of a particular candidate distribution. If the regional average $(\bar{t}_4, \bar{t}_4)$ point falls between two or three distributions whose growth curves are approximately equal, then this implies that two or three models display differences that are statistically significant but operationally unimportant.

Hosking (1986) stated that, for a homogenous region, a probability distribution function is fully characterized by its L-moments, i.e., the weighted average L-moment ratios can represent the parent distribution. The appropriate distribution can be selected by plotting the regional average L-skewness and L-kurtosis values along with the L-skewness and L-kurtosis values for a selection of theoretical distributions on a L-skewness – L-kurtosis graph. Hosking & Wallis (1993) recommended a method for the selection of the parent distribution, based on the use of L-moments.

E. Assessment of Regional Flood Frequency Analysis

Hosking & Wallis (1997) recommended an effective tool for establishing the properties of complex statistical procedures such as the regional L-moment algorithm through Monte Carlo simulation. In the course of simulations, quantile estimates for various nonexceedance probabilities are to be calculated. At the mth repetition, let the estimated regional growth curve and site-i quantile estimate for nonexceedance probability F be $\hat{q}_i^{(m)}(F)$ and $Q_i^{(m)}(F)$, respectively. Then, at site i, the relative error of the estimated regional growth curve as an estimator of the at-site growth curve $q_i(F)$ is $\left[ \left( \frac{1}{N_{\text{sim}}} \right) \sum_{m=1}^{N_{\text{sim}}} \left( \hat{q}_i^{(m)}(F) - q_i(F) \right) \right] / q_i(F)$, and the relative error of the quantile estimate for nonexceedance probability F is $\left[ \left( \frac{1}{N_{\text{sim}}} \right) \sum_{m=1}^{N_{\text{sim}}} \left( \hat{Q}_i^{(m)}(F) - Q_i(F) \right) \right] / Q_i(F)$. To approximate the bias and RMSE of the estimators, these quantiles can be averaged over all M repetitions. Thus, the relative bias and relative RMSE can be expressed as percentages of the site-i quantile estimator by:

$$B_i(F) = M^{-1} \sum_{m=1}^{M} \frac{Q_i^{(m)}(F) - Q_i(F)}{Q_i(F)}$$ \hspace{1cm} (22)
\[ R(F) = \left[ M^{-1} \sum_{i=1}^{N} \left( Q_i^R(F) - Q_i(F) \right)^2 \right]^{1/2} \]  

(23)

Then, a summary of the performance of an estimation procedure over all of the sites in the region is to be obtained through computing the regional average relative bias of the estimated quantile (Hosking & Wallis, 1997) through:

\[ B^R(F) = N^{-1} \sum_{i=1}^{N} B_i(F) \]  

(24)

And the regional average absolute relative bias of the estimated quantile through:

\[ A^R(F) = N^{-1} \sum_{i=1}^{N} |B_i(F)| \]  

(25)

Furthermore, the regional average relative RMSE of the estimated quantile through:

\[ R^R(F) = N^{-1} \sum_{i=1}^{N} R_i(F) \]  

(26)

The regional average relative bias measures the tendency of quantile estimates to be uniformly too high or too low across the whole region. This tendency is apparent, for example, when a distribution with a heavy upper tail is fitted to a region where the true frequency distributions have relatively light upper tails, or vice versa.

The regional average absolute relative bias measures the tendency of quantile estimates to be consistently high at some sites and low at others. This occurs in a heterogeneous region, where the estimated regional growth curve tends to overestimate the true at-site growth curve at some sites and to underestimate it at others. Thus, in a homogenous region, the bias is expected to be the same at each site, and, therefore, \( A^R(F) \) and \( B^R(F) \) are equal (Hosking & Wallis, 1997).

The regional average relative RMSE measures the overall deviation of estimated quantiles from true quantiles. Thus, it is the criterion which gives the most weight for one to judge whether one estimation procedure is superior to another. In addition to the overall accuracy measures of quantile estimates, the corresponding quantities for each site’s growth curve estimate can also be defined by Eqs. (20)-(24) but with \( Q_i(F) \) and \( Q^R_i(F) \) replaced by \( q_i(F) \) and \( q^R_i(F) \), respectively. Comparison of the accuracy of the estimated growth curve and the estimated quantiles facilitates judgment of the relative importance of errors in estimating the index flood and errors in estimating the regional growth curve. Accuracy measures for the growth curve are also relevant when only the growth curve estimate is of interest. The number of repetitions, \( M \), of the simulation procedure must be large enough so that the bias and RMSE measures \( B_i(F) \) and \( R_i(F) \) are close to the true bias and RMSE. This enables reliable comparisons to be made between the performance measures for different regions.

ACKNOWLEDGMENT

At the very first, I’m honored to express my deepest gratitude to my supervisors, Prof. Feng Ping, with whose able guidance and helpful patience and kindness are greatly appreciated. I will also thank Dr. Li Jian Zhu for his support, comments and suggestions.

What’s more, I wish to extend my thanks to Wei Zhao Zhen is a Student in master of science – hydrology and water resources – in Tianjin University for her help.

REFERENCES