Simulation of flow in lid driven cavity by MRT and SRT

Mohammad. Razzaghian, Mohammad. Pourtousi and Amer. Nordin Darus

Abstract— In this paper lid driven Cavity is used to study accuracy and stability between Multi-relaxation-time (MRT-D2Q9) and Single-relaxation-time (SRT-D2Q9). So square cavity has been used for this case study. This simulation is developed for different Reynolds number from 100 to 3200. This model is validated by the numerical simulation of a classical benchmark problem. The results demonstrate the accuracy and effectiveness of the proposed methodology.

Keywords— lid driven cavity; Multi-relaxation-time; Single-relaxation-time; Reynolds number

I. INTRODUCTION

Lattice Boltzmann method with Bhatnagar-Gross-Krook collision model (LBGK)[2,5,6,9] also called single-relaxation-time (SRT) LB method has gotten noticeable success in simulation of hydrodynamic issues. Main important advantages of the LBM can be classified as its explicitly, simplicity performance, and natural to parallelize. There are many other factors that are addressed for improving the lattice Boltzman method such as capability for complex geometry[1,8,9,12], applying of the boundary conditions[4,11,13,14] and also simulation of high Reynolds number. However, despite the presented advantages, some weakness points of the LBGK model are clear. For example this method may cause to numerical instability when the dimensionless relaxation time $\tau$ is close to 0.5. One way for solving these weakness points of the LBGK model is to use a multi-relaxation-time (MRT), which has been shown to make stable solution for high Reynolds number flows[3,7,10] and has the simplicity and computational of the LBGK method. One the main advantages of MRT-LB is that it has better numerical stability and also has more degrees of freedom rather than SRT-LB model.

II. NUMERICAL METHOD

A. Multi Relaxation time Lattice Boltzmann model

In multi-Relaxation method the collision operator is clearly classified as

$$f_i(x + c_\Delta t, t + \Delta t) - f_i(x, t) = -\Omega [f_i(x, t) - f^m_i(x, t)]$$  \hspace{1cm} (1)

In this equation $\Omega$ is the collision step and this term is changed to momentum space and illustrated as

$$f_i(x + c_\Delta t, t + \Delta t) = f_i(x, t) - M^{-1}S[m - m^m]$$ \hspace{1cm} (2)

Where $M$ is a matrix that transforms the distribution function $f$ to the velocity moment $Mf$, and $S$ is the relaxation matrix. The $M$ matrix and its inverse for two dimensional and nine velocity model are

$$M = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\
4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\
0 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\
0 & 0 & -2 & 0 & 2 & 1 & 1 & 1 & 1 \\
0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 
\end{bmatrix}$$ \hspace{1cm} (3)

$$M^{-1} = \beta \begin{bmatrix}
4 & -4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & -1 & -2 & 6 & -6 & 0 & 0 & 9 & 0 \\
4 & -1 & -2 & 0 & 0 & 6 & -6 & -9 & 0 \\
4 & -1 & -2 & -6 & 6 & 0 & 0 & 9 & 0 \\
4 & -1 & -2 & 0 & 0 & -6 & 6 & -9 & 0 \\
4 & 2 & 1 & 6 & 3 & 6 & 3 & 0 & 9 \\
4 & 2 & 1 & -6 & -3 & 6 & 3 & 0 & -9 \\
4 & 2 & 1 & -6 & -3 & -6 & -3 & 0 & 9 \\
4 & 2 & 1 & 6 & 3 & -6 & -3 & 0 & -9 
\end{bmatrix}$$ \hspace{1cm} (4)
Where $\beta = 1/36$ the moment vector $m$ is $m=(\rho, e, j_x, j_y, q_x, q_y, p_{xx}, p_{xy})^T$. The equilibrium of the moment $m^{eq}$ is $m_0^{eq} = \rho$, $m_1^{eq} = -2\rho + 3(j_x^2 + j_y^2)$, $m_2^{eq} = \rho - 3(j_x^2 + j_y^2)$, $m_3^{eq} = j_x$, $m_4^{eq} = -j_x$, $m_5^{eq} = j_y$, $m_6^{eq} = -j_y$, $m_7^{eq} = (j_x^2 - j_y^2)$, $m_8^{eq} = j_xj_y$, where

$$j_x = \rho u_x = \sum_i f^{eq}_i c_{ix}, \quad j_y = \rho u_y = \sum_i f^{eq}_i c_{iy}$$

The diagonal matrix $S$ is expressed as follow

$$S = \begin{pmatrix}
S_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & S_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & S_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & S_4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & S_5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & S_6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & S_7 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & S_8 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_9
\end{pmatrix}$$

In compact notation $S$ can be written as $S = \text{diag}(1, 1.4, 1.4, S_3, 1.2, S_4, 1.2, S_7, S_8)$ where $S_1 = S_8 = 2/(1 + 6\nu)$ and $S_3$ and $S_7$ are arbitrary which can be set as 1.

Finally the macroscopic variables such as the density and fluid velocity can be computed in terms of the particle distribution functions as

$$\rho = \int_i f_i dc, \quad \rho u = \int_i c_i f_i dc$$

(7)

Figure 1 shows the geometry case study and the boundary condition;

B. Single Relaxation time Lattice Boltzmann model

The structure of the single relaxation time scheme is simple and the particle distributions relax to their local equilibrium at a rate determined by single parameters. The D2Q9 model has been implemented for this purpose. For the incompressible problems, the evolutions of particle distribution functions are computed by the following equations:

$$f_i(x + c_i\Delta x, t + \Delta t) - f_i(x, t) = \frac{1}{\tau}\left[f^{eq}_i(x, t) - f^{eq}_i(x, t)\right] + \Delta tF_i$$

(8)

Where the density distribution function $f$ is used to compute the density and velocity fields. $\Delta t$ represents lattice time step $c_i$ is the discrete lattice velocity and $F$ is the external force in direction $i$, $\tau$ represents the relaxation time for the flow. The $f^{eq}$ in Eq. (8) is equilibrium distribution function for density. The configurations of the lattice velocities is shown in Fig 2.

The discretised equilibrium distribution function for D2Q9 is given as

$$f^{eq}_i = \omega_i \rho \left[1 + \frac{c_i \mu}{2 c_s^2} + \frac{(c_i \mu)^2}{2 c_s^4} - \frac{1}{2} \frac{u^2}{c_s^2}\right]$$

(9)

Where $\omega_0 = 4/9$, $\omega_{1-4} = 1/9$, $\omega_{5-9} = 1/36$ and $c_s$ is the speed that it equals $\sqrt{3}/3$ while $\Delta x = \Delta t = 1$

III. RESULTS AND DISCUSSION

A. Influence of Reynolds number

In this letter different Reynolds numbers (100, 400, 1000 and 3200) are used in order to analyze which method of Lattice Boltzmann (LBM) has much accuracy. Furthermore velocity in X and Y direction is validated by Ghia in both method.

95
This study has proved accuracy of MRT in comparison by SRT which is represented in Figure 3.1 and 3.2. Moreover, it has to be noted that Figure 3.1 and 3.2 show that MRT has a good agreement with Ghia result in preview literatures.

By increasing the Reynolds number MRT has emerged as an accurate method apart from SRT, figure (3.3) and (3.4) is represented accuracy of MRT for Re 400.

It is interesting to say that by growing Reynolds number SRT will be approached to instability for example in Re = 1000 SRT is not stable with low mesh. This investigation is presented lid driven cavity by 100*100 mesh for Reynolds 1000.

Figure (3.8) is shown that 100*100 meshes are used for this lid driven cavity and furthermore it should be noted that these meshes have a good stability till Re=1000 for MRT.

IV. CONCLUSION

Square cavity has been used for this study. The movement wall is considered as a movement boundary condition but bounce-back boundary condition shows stationary wall. Multi relaxation time in both accuracy and stability has been shown in this work. On the other hand SRT will be stable for 400 and after this number the code will be crashed. In addition, these Results are validated excellent agreement with exist numerical results.

ACKNOWLEDGMENT

The authors would like to thank Prof. Amer Nordin Darus for helpful discussions.
REFERENCES


