Mathematical model of time dependent of Persian gulf circulation

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Abstract—Time dependent ocean circulation is studied in a simple half-bounded-ocean (Persian Gulf). The model solves the vorticity equation which includes pressure gradient force, Coriolis force, friction force and component of unsteady wind stress. The ocean model used is for uniform incompressible and homogeneous sheet of semi-confined fluid. Both the linear response and the steady non-linear circulation are considered. First the free oscillation (normal mode) are considered and results show that each normal mode include a travelling wave (with phase speed always toward the west), and circulation are cells of vertical motion. The resulting cell move westward, alternatively increasing and decreasing in size. The equation is solved with the effect of forcing function which consists of a travelling wave over the ocean in three cases. Case one, when forcing frequency lies within the range of the characteristic frequencies of the ocean basin. In this case, linear response includes four normal modes that each mode have a phase different of 2π, with the adjacent mode, and the non-linear response in first mode, consists of two cells. One north of \( y = l / 2 \), the other south of \( y = l / 2 \). The circulation in the southern cell is counter clockwise and that of north cell is clockwise. This phenomena seem to occur in the months of August and March. In case two, the forcing frequency is much larger than the frequency of normal mode. The linear response in the form of reflection in coastal boundaries, and felt everywhere in the ocean interior. In non-linear response, circulation consists of two cells. The direction of circulation in cells are different. This event seem to appear in all months. Case three, when forcing frequency is smaller than the normal mode frequency, linear response is an oscillating boundary layer in the coastal boundaries and in another boundaries it is a travelling wave moving in the direction of forcing wave. Non-linear response consists of two cells. These cases do not seem to occur in any month of the year. Thus, the symmetry, structure, sense, and strength of circulation and strongly frequency dependent. Although, the results seem to be consistent with the observation, but one requires to use a more realistic model such as the numerical ones including possibly the baroclinicity. (e.g. a two-layer).

Keywords—circulation, mathematical model, time dependent.

I. INTRODUCTION

Up to now, the study of ocean circulation in a closed domain, has been limited to the analysis of reaction of different oceanic model on steady wind stress force. For instance, Stomel [20] in 1948, conducted a study on water circulation in a rectangular and homogeneous ocean that is caused by wind stress, bed friction, Coriolis force and horizontal pressure gradient, according to diversity of height.

He realized that the reason of density, compression, concentration of current lines in western part is variation Coriolis parameter with latitude. Munk [13] in 1950 studied that mass transfer of oceanic current lines using vertical integral equation of curl that includes planetary curl, lateral stress curl, wind stress curl.

Cherney [5] investigated in 1955, oceanic circulation considering boundary layer fluid, only with the presence of internal and pressure forces. In all of the examples, it is assumed that the wind force has been considered steady, while it is known wind force, especially in middle latitudes is unsteady. On the other hand, an interesting numerical study, however not very accurate on unsteady wind stress was conducted by Veronic [23] in 1963.

He provided motion equations for a square closed ocean, his equations included time dependent, non-linear effects, wind stress and line variation of Coriolis parameter. He concluded that ocean cannot be steady at all, however after transition from an unsteady primary phase, it arrives to a limited periodic circle.

In this research, unsteady wind stress component is discussed, in addition to answering the ocean oscillating, it has investigated wind force oscillation with zero time average, that leads a steady time independent circulation and follows non-linear dynamic rules.

This paper is about possible oscillations and steady circulation made by time dependent of wind force in a simple model is discussed and particularly investigated dependence of circulation resulted with parameters of Persian gulf internal and force.

II. GEOGRAPHY

Persian gulf, a semi-closed sea, with a length of 1000 km, and a width of 200-300 km, i.e. a domain about 226000 km². The average depth of the gulf is about 35 m with a maximum depth of 100 m in Hormoz strait, with an estimated volume 860 km³.

Persian gulf is between geographic latitudes 24° to 30°, and 3° of North and longitudes 48° to 57° Eastern.

In the South, there are ten kilometers of dry and flat deserts. Fluvial currents in Western South Coast of Persian gulf are limited to random floods that are loaded by local storms. In the north of the Persian gulf, rainfalls due to Mont. Zagros...
ranges, create fluvial currents, in a way that the majority of rivers which flow into Persian gulf, are in northern and Iranian part. Generally, Persian gulf has a dry and half-warm weather.

III. PERSIAN GULF: WIND

In summary, effective parameters on Persian gulf wind regime are:

a) High pressure extensive cores (location of air drive) and low pressure (location of air suction) formed in different seasons.

b) Differences of characteristics of water and soil.

c) Mont. Zagros lead currents along the coast.

d) Mediterranean or Sudanese cyclones and the depended fronts.

Considering such effective parameters, in Persian gulf winds regime, they are divided into four categories according to their local names:

a) North wind: Actually, it is a western north wind that blows in this area during 8 months, from October to May. There is North wind in summer too, however in cold season, the current of water is more intensive.

b) Nashi wind: This is an eastern north wind that blows, especially in Bandarabbas and Hormoz strait areas, it mostly continues 3 to 5 consecutive days. The wind feels in Hormoz Strait and Bandar Abbas.

c) Shaki wind, is called Eastern or Sharji wind, sometimes called Eastern South or sometimes Eastern and drives in Hormoz Strait in winter to mid spring.

d) Soheili wind: It is a local name for Western South wind which just blows in winter. It's duration is very short with rain, heavy shower and thunderstorm.

IV. MODEL

Model includes a uniform, incompressible, homogenic fluid layer. In vertical axis, hydrostatic assumption prevail, therefore, motion is investigated in two different dimensions. There is a change in linear corilious parameter; i.e. equation is written in \( \beta \) Plane.

V. EQUATIONS OF CIRCULATION

Considering assumptions of model, equations of motion in two dimensions are:

\[
\begin{align*}
\frac{\partial U}{\partial t} + u \frac{\partial U}{\partial x} + v \frac{\partial U}{\partial y} &= -1 \frac{\partial p}{\partial x} + fV - \frac{k}{\rho_0} u + \frac{T_x}{\rho_0 H} \\
\frac{\partial V}{\partial t} + u \frac{\partial V}{\partial x} + v \frac{\partial V}{\partial y} &= -1 \frac{\partial p}{\partial y} - fV - \frac{k}{\rho_0} v + \frac{T_y}{\rho_0 H} 
\end{align*}
\]

(1)

(2)

Now, from equation of (1) differential ratio of \( y \) and equation of (2) differential ratio of \( x \) is done, and then minus them. Then \( u \) and \( v \) are written versus current function "\( \Psi \)."

VI. BOUNDARY CONDITION

Because Persian gulf is a semi closed area, boundary conditions are:

\[
\Psi = 0 \quad (x = 0, \quad x = \infty, \quad y = 0, \quad y = l)
\]

VII. PROBLEM SOLUTION

For solving Curl equation, dimensionless variations are firstly introduced.

\[
(x, y) = L(x', y'), \quad t = (\beta L)^{-1} t'
\]

\[
\nabla \times T = \frac{\tau_0}{\rho_0 H L} T(x', y', t), \quad \Psi = \frac{T}{\rho_0 \beta H}
\]

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Secondly, they are located in curl equation:

\[
\nabla^2 \Phi + \Phi x + R. \nabla^2 \Phi = \nabla \times - \nabla \Phi 
\]

To apply boundary layer in educations, it assumes that current function and wind stress force are in form a sinus series.

\[
\Phi = \sum_{n=1}^{\infty} R_{\psi(n)} \sin n \pi y
\]

By locating (7) series in (5) equations, it will yield:

\[
\frac{\partial}{\partial t} (\varphi_{mxx} - n^2 \pi^2 \varphi_n) + \varphi_n + \delta(\varphi_{mxx} - n^2 \pi^2 \varphi_n) = \tau_n
\]
General solution of equation (7) by using Laplace transformation is:

$$\phi_n = \frac{2}{i} \int_{-\infty}^{\infty} e^{-\alpha_n x} \cos \left( \frac{x - x'}{2(s + \delta)} \right) \exp \left( -\frac{\alpha_n x}{S + \delta} \right) dt$$

$$\hat{y}(t) + v_n \hat{x}'(t)$$

Equation (9) is a general solution that should be discussed under certain conditions.

1. **Normal modes**: In this part, water free oscillations \((\tau_n = 0)\) according to equation (9) are discussed. Considering equation (9), the amplitude of Normal modes is:

$$\Phi^{(o)} = e^{i\theta} \begin{bmatrix} \cos \left( \frac{x}{2\omega_n} + \omega_n t \right) \\ \sin \left( \frac{x}{2\omega_n} + \omega_n t \right) \end{bmatrix} \sin n\alpha y$$

As we see, the normal modes include a progressive wave towards west. Circulations in any mode are informing vertical moving cells that are moving toward west and their measure will become small and large alternatively.

2. ** Forced solutions**: Now, equation (6) according to time dependent wind force function is discussed. It is assumed that \( T = \cos(kx - \omega t) \sin n\alpha y \). By placing and solving equation of (8), general solution is:

$$\varphi_n = 4i e^{a_n x} \frac{\sin \alpha_n x}{\alpha_n} \cos(kx - \omega t) \exp \left[ -i \left( \frac{x - x'}{2(\omega + i\delta)} \right) \right]$$

(10)

Although (10) is a general solution, form of solution is variable. Therefore in explaining nonlinear circulations, attention be made to especial solutions.

A. **First state**: Resonance \( \omega = \omega_n \)

This is one of the most important states, it means, when frequency of wind functions in the range of normal modes frequency.

Firstly, linear answer is studied:

$$\Phi^{(o)} = \frac{-2i}{n\pi} \begin{bmatrix} e^{-i\left( \frac{1}{2w} + k \right)x} e^{-i\left( \frac{2}{2w} + 2n\pi \right)x} \\ e^{-i\left( \frac{1}{2w} + k \right)x} e^{-i\left( \frac{1}{w} + 2n\pi \right)x} \end{bmatrix} \sin n\alpha y$$

(11)

As can be seen, it includes four normal modes that frequency difference between two of them is \(2n\pi\).

In overall state when wind force frequency increases, amplitude of linear answer increase too. For creation first nonlinear affections, equation of (6) is referred. Jacobian of current function, zero order of current function should be calculated and especial solution of equation of (6) (nonhomogenous) is generated:

$$\Phi^{(o)} = \frac{2}{n\pi} \begin{bmatrix} \omega A(k,\omega_n) e^{i\left( \frac{x}{\omega} + \frac{k}{\omega} \right)} + B(k,\omega_n) \left( \frac{1}{\omega} + 2k + 2n\pi \right) \\ \frac{1}{\omega} - k + 2n\pi \end{bmatrix}$$

$$+ \frac{c(k,w_n)}{\omega_n} e^{-i\left( \frac{1}{2\omega_n} + 2n\pi \right)x} + \frac{D(k,w_n)}{\omega_n} e^{-2i\left( \frac{1}{2\omega_n} + k + 2n\pi \right)x}$$

$$+ \frac{E(k,\omega_n)}{2(\frac{1}{\omega_n} + 2n\pi)} + \frac{F(k,\omega_n)}{2(\frac{1}{2\omega_n} - k + 2n\pi)}$$

$$e^{-2i\left( \frac{1}{2\omega_n} + k + 2n\pi \right)x} + G(k,\omega_n) \sin 2n\pi y$$

Andis (S) means that function is time independent. First mode \(n=1\) includes two cells, one is North \(y=L/2\) and another is South \(y=L/2\), circulation in South cell is antilockwise. States of these circulations are nonhomogeneous and dependent to structure of normal modes and independent to parameters of force. Interesting point is that \(\Phi^{(o)}\) not only corrects in condition that current is zero in \(x=\infty\) and \(x=0\), but also tension velocity is zero in both West and East boundaries, and it is the advantage of this solution.

B. **Second state**: \( \omega = \omega_n \quad \omega >> \delta \)

When force frequency is very larger than time range of normal modes, Linear solution for least order of \( \frac{\delta}{\omega} \) is:

$$\Phi^{(o)} = 4i e^{a_n x} \frac{\sin \alpha_n x}{\alpha_n} \cos(kx - \omega t) \exp \left( -i \left( \frac{x}{2\omega} \right) \right) \sin n\alpha y$$

(13)

(12) indicates a moving wave toward west that its phase is equal wind force's phase, if \( \omega < \frac{1}{2n\pi} \), \( \alpha_n \) is imaginary and the modulated functions are of oscillatory nature and wave reflection of coastal boundaries with wind force wave interfere in every place in Persian gulf. On the other hand, in high frequency, when \( \omega > \frac{1}{2n\pi} \), \( \alpha_n \) is real and boundaries reflection, especially when \( n \) is large, far from boundaries exponentially damps.

Nonlinear steady circulation is:
It is clear that in high frequency, there is a steady boundary layer in one side of Persian gulf, for \( n=1 \), again, solution includes two cells, one in the upper and another in the lower half. Circulation states in every cell are inverse of resonance state. Current in every cell is specific to that area and far from the boundaries.

In near the boundary, current is intensively turbulent.

**C. Third state:** \( \omega << \frac{1}{2n \pi} \) \quad \omega = o(\delta)

When force time scale is very larger than normal modes time scale, special form of linear solution of (5) is: (According to \( S^2 = 2(\delta^2 + \omega^2) \))

\[
\Phi^{(o)} = 4\left(\frac{S^2}{2\delta}\right)\sin\left(\frac{-2x \delta}{S^2}\right)\exp\left(\frac{2\delta}{S^2}(x - 1)\right)
\]

\[
\cos(kx - wt)\sin(n \pi y)
\]

Time part of this solution indicates a progressing wave in the direction of force wave, while static part demonstrates an oscillatory boundary layer, this section is just in west side of ocean. Now, nonlinear order is explained first. For algebraic simplification, a special solution is choosen it is assumed \( (S^2 = 4\delta^2 , \frac{\delta}{s^c} << 1) \)

therefore:

\[
\Phi^{(1)}_x = -64 \exp\left(\frac{x-1}{2\delta}\right)\{k \cos^2\left(\frac{x}{2\delta}\right)\sin 2kx\}\sin 2n\pi y
\]

\[
\Phi^{(1)} = -64\{3 / 6k\left[\frac{1}{1+4k^2} + \frac{1}{-3+4(i+k^2)} + \frac{1}{-3+4(k^2-i)}\right] + 2i \exp\left(\frac{x-1}{2\delta}\right)
\]

This steady current includes 4 parts. First, indicates a constant value and the solution is not oscillatory. Second part, there is a boundary layer thickness twice as much the Linear Steady that after \( \delta^{-1} \) time is damped. In third and forth section, there is a thickness Linear steady of boundary layer that is oscillatory alternative. For \( n=1 \), the tangential velocity is negative in western boundary in the lower half of the ocean and its magnitude reduces exponentially. Therefore, there is an anticlockwise rotating cell in ocean. Lower half and a clockwise rotating cell in ocean upper half, that is compressed in a very thin area.

**VII. SOLUTION OF EQUATION WITH PERSIAN GULF PARAMETER**

In this section, Persian gulf parameters is applied for three states that is solved in last step and \( \Phi^{(0)}, \Phi = \Phi^{(0)} + \Phi^{(1)} \) is drawn.

**A. First state:** \( \omega = \frac{1}{2\pi n} \) \quad \omega = \omega n

\( (n = 1) \quad \omega_1 = 0'16 \quad V = 5 \quad k = \frac{\omega}{v} = 0'032 \)

\[
\Phi^{(o)} = 4144 / 7 \sin 2\pi x \left[\cos(0 / 16t + 3 / 125x) - \cos(0 / 16t + 3 / 125(x - 1)) - 0 / 03y \sin ny\right]
\]

\[
\Phi^{(1)} = -9587379923\sin^2\pi x \sin 2\pi y \{1 - \cos(3 / 152)\}
\]

**Fig1. Drawing Current Function \( \Phi = \Phi^{(0)} + \Phi^{(1)} \)**

To draw zero order current function in five different times, shows the most changes.
Second state: $\omega >> \delta$

$\nu_{\text{max}} = 25\text{Hz}$ \hspace{1cm} $\omega = 157/08$ \hspace{1cm} $V = 5\frac{m}{s}$

$k = 31/41$ \hspace{1cm} $\alpha = 10.5$

$\Phi^{(o)} = 6/45 \times 10^{-6} \left[ \sin(31/42 \times 157/08 - 4/184) \right]$

$\sinh 10/5 (x - 1) + \sin(157/08 + 4/184(x - 1) -$

$\frac{\sinh 10/5}{\sinh 10/5} + \sinh 10/5$

$\Phi^{(b)} = 1/03 \times 10^{-6} \sin 2\varphi[1 - 0.04321391\varphi^6]$

$\Phi = \Phi^{(o)} + \Phi^{(b)}$

Fig. 2 Drawing Current Function $\Phi = \Phi^{(o)} + \Phi^{(b)}$

To draw zero order current function in five different times, shows the most changes.
Third state: \( \omega = \omega(\delta) \)
\[
\delta = 0.05 \quad \omega = 0.05 \quad V = 5 \quad K = 0.01
\]
\[
\Phi^{(o)} = [(x - 1) + e^{-20x}] \sin \pi y
\]
\[
\Phi^{(1)} = [(\cos 10x - \sin 10x)e^{-20x} - (20x + 1)e^{-20x}]
\]
\[
157 / 0796326 \sin 2\pi y
\]
\[
\Phi = \Phi^{(o)} + \Phi^{(1)}
\]

Fig.3 Drawing Current Function \( \Phi = \Phi^{(o)} + \Phi^{(1)} \)

To draw zero order current function in five different times, shows the most changes

V. CONCLUSION

As said in introduction for researching Persian gulf circulation completely and precisely, complete spectrum of wind force is necessary, however in this paper just average wind velocity is assumed. Because, Persian gulf has two layers, non-uniformity should be paid attention to. On the other hand, considering the low depth of Persian gulf, flat friction should be written and X axis should be in Persian gulf axis direction.

Although the model is simplified, it's conclusions are completely compatible with, Alhajeri model, just available model. Other models are not available.

In summary, variety of state in Persian gulf circulation structure intensively is related to frequency of wind force.

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Application of cutting-edge social media technologies in Connectivist Pedagogy

Dr. Charles Kivunja

Abstract—Although the penetration of social media technologies into orthodoxy teaching and learning has been slow, there is an increasing awareness that embedding social media in pedagogy bestows upon students much greater capacity to construct knowledge according to Piaget, co-construct understanding through interaction with others as postulated by Vygotsky; effectively utilizes Brunner's 5E Instructional Model; is consistent with Bloom's Cognitive Taxonomy; applies DeBono's Six Thinking Hats problem solving strategies and creates opportunities for students to learn in their styles as postulated by Howard Gardner's Multiple Intelligences.

This paper reports the lived experiences encountered when embedding Google+, Discussion Circles, eFoliospaces and YouTube social media technologies in connectivist pedagogy for doctoral students and 2nd Year B-Education pre-service teachers at a University in Australia.

The experiences reveal that the introduction of selected social media technologies into pedagogical practice has potential to shift learning paradigms from Constructivism to Connectivism and from Instructional to ultra-modern Digital Pedagogy.

Keywords— Connectivism, Constructionism, Constructivist Pedagogy, Digital immigrants, Digital nativity, Digital Pedagogy, eFoliospaces, Google+, Discussion Circles, Social media technologies, Social media technophobia, Student-led inquiry.

I. INTRODUCTION

It is well documented that the advent of social media sites such as Facebook, Twitter, MySpace and Bebo was warmly embraced for application in recreational and conversational interaction, mainly for fun and humour [1]. However, their penetration into orthodoxy education settings where the rigors of academia are shrouded in competition and survival of the fittest, has been more elusive and sometimes controversial. Drawing from personal experiences at one of my former schools, whereas we encouraged our students to use computers to develop competitive models of profit maximisation in Economics, we blocked their access to social media. Moreover, especially in the lower stages, students were allowed to use computers only if they had completed their set learning activities. In particular, the thought of secondary school students, including year 12 high school students turning 18 years old, accessing ‘social media’ exposed to global audiences and capable of instantaneous responses, sent trepidation and shivers down our spines as we implemented what amounted to covert censorship and surveillance measures in our efforts to ‘protect’ the students from what we regarded as potentially distractive or harmful sites. Thus, access to social media was not seen at part of standard pedagogical practice. As a result, students were denied access to digital learning resources, which could have been helpful in their construction of knowledge about High School Economics. This social media technophobia seems to sustain the gulf between, on one side, academics (and teachers) who advocate reliance on orthodoxy textology-digital immigrants- and (on the other) those inclined to towards embracing and extending the footprint of digital nativity. Whereas it is well argued that it is difficult for institutions like schools and universities to manage the use of social media by their members [2], it is suggested in this paper that this technophobia is unwarranted and not only needs to be exposed, but worked upon to be rectified. What’s more, the paper argues, that the introduction of selected social media technologies, as part of well-structured, computer-supported collaborative epistemology, into pedagogical practice has potential to shift learning paradigms from Constructivism to Constructionism [3]; from Constructionism to Connectivism [4] and from Orthodoxy Instruction to Digital Pedagogy [5].

One way to realize this potential, is to encourage pre-service teachers as well as present and future educational leaders [6] to see the potentially great value of social media as highly motivational, interesting inquiry-based, student-led learning that has the potential to shift our pedagogical practice paradigm from Vygotsky’s [7] constructivist pedagogy, to constructionist pedagogy [3] and connectivism, as we move our teachers and students towards becoming digital natives. Such social media have great value not only in Best Practice Pedagogy in the classroom, but also in workplaces, be they institutions of higher learning or otherwise. Relying on the theoretical constructs of giants in the field of Best Practice Pedagogy, - giants upon whose shoulders we stand as we try to make our own contribution to pedagogical practice, - convinces us that the application of social media technologies in teaching and learning can only have meritorious consequences on knowledge creation by our students. For example, Vygotsky [7] tells us that children learn best when they are actively engaged in the construction of knowledge through social interaction with others. Social media technologies appear to excel in this regard. A second example is taken from Brunner [8] whose 5E Instructional Model

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