Abstract - A new sliding surface design approach for magnetic levitation system is proposed that varies the sliding surface in a nonlinear and time-varying fashion. The control law is designed by using the surface that is defined in a new co-ordinate axis. The nonlinear surface is then moved in a proper direction by using a time-varying function. Simulations have been performed on a second-order nonlinear model of a magnetic levitation system. The result of the new design method is compared with the classical sliding mode controller possessing a discretely moving sliding surface. From the results we can conclude that the reaching time has considerably reduced and robustness to disturbance has increased. Also the phase plane portrait has much smoother trajectories.

Keywords--- Magnetic levitation system, nonlinear time varying sliding surface, sliding mode control, sliding surface design

I. INTRODUCTION

MAGNETIC levitation systems (MLS) have realistic importance in many engineering systems such as in high-speed maglev trains, frictionless bearings, levitation of wind tunnel models, and vibration isolation of sensitive machinery, levitation of molten metal in induction furnaces, and levitation of metal slabs during manufacturing. The maglev systems can be classified as attractive systems or repulsive systems based on the source of levitation forces. These systems are usually open-loop unstable and are described by highly nonlinear differential equations which present additional difficulties in controlling these systems. This unstable aspect of MLS and its inherent nonlinearities make the modeling and control problems very challenging. Therefore, it is an important task to construct high-performance feedback controllers for regulating the position of the levitated object.

During the last 20 years, sliding mode control (SMC) has received significant interest and has become well-established research areas for practical applications. The discontinuous nature of the control law in SMC results in outstanding robust-ness features for both system stabilization and output tracking problems. The good performance also includes insensitivity to parameter variations and rejection of disturbances. SMC has been applied in many control fields which include robot control, motor control, flight control, control of power systems, and process control. The first application of SMC to magnetic levitation systems was carried out by Cho et al [3]. It was shown that a sliding mode controller provides a better transient response than classical controllers. However, current dynamics was neglected in their model and limited the ball’s motion to a range of 1 mm. Chen et al [2] designed an adaptive sliding mode controller for a rather different type of magnetic levitation system called dual-axis maglev positioning system. Buckner [1] introduced a procedure for estimating the uncertainty bounds using artificial neural network and then applied it to SMC of a magnetic levitation system. Hassan and Mohamed [4] used the reaching law method complemented with the sliding mode equivalence technique to design a variable structure controller for the magnetic levitation system. The main steps of designing a SMC are mainly 1) Determining sliding surface that governs the system dynamics 2) A control law that makes state trajectories approaches the sliding surface.

The phase trajectories of an SMC represent two modes of the system [7]. The trajectories starting from a given initial condition off the sliding surface moves towards the sliding surface. This is known as reaching phase, and the system is sensitive to parameter variations in this part of the phase trajectory [6]. When the convergence to the sliding surface take places, the sliding phase starts. In this phase, the trajectories are insensitive to parameter variations and disturbances. [8]. To improve the performance of SMC, two well known sliding surface design methods are used. They are 1) Time varying scheme for constant linear sliding surface 2) Non-linear sliding surface

In time varying scheme, sliding surface is shifted and rotated in the state space to improve the tracking problem. This design method is easy but may have performance disadvantages with respect to nonlinear methods. The magnitude of control law required to keep the states on the sliding surface usually increases as the magnitude of the tracking error increases. Thus we need different dynamic
properties in different times which are not achieved by time varying method. Non-linear sliding surface design method provides a wide variety of design alternatives form the linear counterpart, since a large scale of static relationship can be synchronized when non-linear sliding surface values are used in SMC. A disadvantage of this scheme is that finding non-linear function in these studies has analytical difficulties and it is complicated to define parameters of the non-linear function.

In this study we present a new approach as an attempt to combine the benefits of both the scheme of adoptive linear sliding surface and nonlinear sliding surface at the same time. The proposed approach varies the sliding surface in a nonlinear and time varying fashion. The proposed controller is based on the SMC schemes developed by S. Tokat, I. Eksin and M. Guzelkaya [5]. It is designed on new co-ordinate axes, one of which is classical sliding surface and other is naturally chosen to be orthogonal to it. The nonlinear sliding surface is then moved in a proper direction by using a time varying function. Simulations are performed on a second order nonlinear magnetic levitation system with bounded external disturbances. This method has improved the system performance in terms of a decrease in reaching time robustness to disturbances and a smoother phase plane trajectory when compared to its counterparts.

The rest of the paper is organized as follows. Section II contains the mathematical model of the magnetic levitation system. Section III deals with the design of a classical SMC for the magnetic levitation system. Sections IV deal with the design of a new sliding mode controller using nonlinear time varying sliding surface as proposed in [5] for the system. Section V presents and discusses the simulation results of the proposed control schemes. Finally, the conclusion is given in Section VI.

II. MAGNETIC LEVITATION SYSTEM

Magnetic levitation system considered in the current analysis is consisting of a ferromagnetic ball suspended in a voltage-controlled magnetic field. Coil acts as electromagnetic actuator, while an optoelectronic sensor determines the position of the ferromagnetic ball. By regulating the electric current in the circuit through a controller, the electromagnetic force can be adjusted to be equal to the weight of the steel ball, thus the ball will levitate in an equilibrium state. But it is a nonlinear, open loop, unstable system that demands a good dynamic model and a stabilized controller.

Free body diagram of ferromagnetic ball suspended by balancing the electromagnetic force \( f_{em}(x, i) \) and gravitational force \( f_g \) is shown in Fig. 1. The equivalent mathematical model of the closed loop control system is expressed as:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{2ci_o}{m_0^3} x - \frac{2ci_o}{m_0^2} u
\end{align*}
\]

Where, \( u \) is applied voltage, \( i \) is current in the coil of electromagnet, \( R \) is coil’s resistance and \( L \) is coil’s inductance, \( m \) is mass of ball, \( x \) is position of the ball, \( g \) is gravitational constant and \( c \) is magnetic force constant. This is the mathematical model we will use for our simulation purpose.

![Fig. 1 Free body diagram of magnetic levitation system](image)

III. CLASSICAL SLIDING MODE CONTROL

A single input open-loop system of order \( n \) can be given as

\[
x^{(n)}(t) = f(x, t) + b(x, t)u(t) + d(x, t) (2)
\]

Where \( x^{(n)}(t) = [x, \dot{x}, \ldots, x^{n-1}] \) is the state vector, \( u(t) \) is the input signal, \( d(x, t) \) is a time dependent disturbance with known upper bound, and \( f(x, t) \) and \( b(x, t) \) are functions determining the system dynamics. For single-input systems, the commonly used sliding surface can be defined as

\[
S(x, t) = Ce(t)
\]

Where \( C = [c_{n-1}, c_{n-2}, \ldots, c_1] \in \mathbb{R}^{1 \times n} \) is a vector with strictly positive real elements that determine the coefficients of the sliding surface and \( e(t) \) is the tracking error defined as

\[
e(t) = x(t) - x_d(t)
\]

Where \( x_d(t) \) is the desired trajectory. In this study, second-order systems are considered. Therefore, (4) can be written as

\[
S(x, t) = e(t) + C_1 e(t)
\]

This gives a linear function in terms of error with a slope value \( C_1 \). The error will asymptotically reach to zero with an appropriate control law that could keep the trajectory on the sliding surface. Lyapunov’s direct method could be used to obtain the control law that would solve this problem of keeping \( s(x, t) \) at zero and a candidate function can be defined as

\[
V(s) = \frac{1}{2} S^2(x, t)
\]

With \( V(0) = 0, V(s) > 0 \) for \( \forall S(x, t) > 0 \) An efficient condition for the stability of the system can be given as

\[
\dot{V}(s) = SS' \leq -\eta |S(x, t)|
\]

Where \( \eta \) is a strictly positive real constant. By substituting (2) into (7) and omitting the argument of the independent variables, one obtains

\[
S(f + b + u + d - \dot{x}_d + C_1 \dot{e}) \leq -\eta |S|
\]

Therefore, a control input satisfying the reaching condition can be chosen as

\[
u = \frac{-f + x_d + C_1 \dot{e}}{b} - K \text{sign}(S)
\]
$K$ is a strictly positive real constant with a lower bound depending on the estimated system parameters and bounded external disturbances. The function \( \text{sign}(\cdot) \) denotes the signum function. The signum function denotes the discontinuous part of control law represented by \( u_d \). The continuous part of control law is known as equivalent control and is denoted as \( u_{eq} \).

IV. TIME VARYING NONLINEAR SMC

The control law (9) may be sensitive to uncertainties and disturbances acting on the system during the reaching phase. In this paper, the control law (9) is adjusted by using a nonlinear and time varying sliding surface. We develop a new plane within the original \((e - \dot{e})\) phase plane that is shown in Fig. 2. One of the coordinates is taken as the original sliding surface \( S \). The other coordinate is normally a perpendicular line to \( S \) and it is described as

$$ P = -(1/c) e + \dot{e} \quad (10) $$

This variable is used to generate a new sliding surface in the \((s-p)\) coordinates. By using Equation (10), a nonlinear parabolic sliding surface can be suggested as

$$ \sigma = S - \varphi P^2 \quad (11) $$

Here, \( \varphi \) gives the position of the parabolic surface. For positive values of \( \varphi \), the nonlinear surface given in Equation (11) is over the classical sliding surface defined in Equation (5) and for negative values it bends to the opposite side as shown in Fig. 2. This property can be used to alter the discontinuous control law and to affect the system states. In order to obtain the discontinuous control law given in Equation (9), the value of the sliding surface is calculated using Equation (11) instead of the classical linear constant surface function. Thus, a nonlinear rotating sliding surface is obtained by changing the value of \( \varphi \) continuously in time.

The minimum and maximum allowable values of \( \varphi \) are \( \varphi^- \) and \( \varphi^+ \) respectively. By substituting the new sliding surface equation (11) into the Lyapunov equation in (6) and the reachability condition in (7), the control law can be reformulated as

$$ u = \frac{\left[ -\left( c_1 + \frac{2\varphi P}{c_1} \right) \dot{e} + \varphi P^2 - K \text{sign}(\sigma) \right] - f + \ddot{x}_d}{b} \quad (13) $$

Reachability condition is satisfied for inequality given as \((1 - 2\varphi P) \geq 1\).

V. SIMULATION RESULTS

The simulations are performed on a non-linear magnetic levitation system whose mathematical model is given by equation (1), whose physical parameters are given as \( c=0.00023142 \ \text{Nm}^2/\text{A}^2, i_0 = 0.6105 \ \text{A}, \ x_0 = 20 \ \text{mm}, \ R=13.8 \ \Omega \). The physical allowable operating region of the steel ball is limited to \([-10 \text{mm} \leq x_t \leq 10 \text{mm}] \). The velocity \( x_t \) is measured by pseudo-differentiation of the measured position \( x_t \) as \( \dot{x}_t/(0.001s + 1) \). The initial conditions are taken as \( x_t(0) = 0.01 \text{m} \) and \( x_t(0) = 0 \text{ m/s} \). First the simulations are performed for the classical SMC with a constant sliding surface and then proposed SMC with nonlinear time varying sliding surface. For the classical SMC, taking \( C_1 = 5 \) and \( K = 2 \). The control input is obtained as

![Fig. 3 control input for classical SMC](image)

The proposed SMC is implemented by choosing \( \varphi = 0 \), \( C_1 = 5 \) and \( K = 2 \). The parameters that determine the shape of \( \varphi \) are chosen as \( m = 2.2, \alpha = 0, \varphi^- = -3 \) and \( \varphi^+ = 0.4 \). The control input is obtained as

![Fig. 4 control input for nonlinear time varying SMC](image)
After reaching the sliding phase the phase plane trajectory is nonlinear and it smoothly enters the sliding phase in the proposed method. However, the classical SMC has a linear sliding line and the effect of the disturbance can be easily seen from the phase plane trajectory. From Fig. 3 and 4, it is seen that the structure of the new sliding surface decreases the reaching time and therefore decreases the effect of disturbances. The reaching time for classical SMC is 4 seconds, while for proposed SMC it is 0.56 seconds. The chattering can be reduced by smoothing the control action across the sliding surface using a saturation function instead of a signum function in the discontinuous control part.

VI. CONCLUSIONS

In this study, a new nonlinear sliding surface with parabolic and time-varying characteristics is proposed to improve the discontinuous control law of a classical SMC, by combining the benefits of both the viable scheme of moving linear sliding surfaces and successful but complex implementation of nonlinear sliding surfaces at the same time. The results have shown that the proposed method improves the system transient response by minimizing the settling time, and lessens the negative effect of disturbances as it decreases the reaching time. For a classical SMC, there is a trade-off between reaching time and settling time. Therefore, when one of them is improved, the other one gets worse in return. The states enter smoothly into the sliding phase in the proposed SMC. This is the main advantage of the proposed mechanism. Here, in this study, the bending of the sliding surface is obtained by a simple time dependent function. This function can also be obtained through system-dependent algorithms or relations to increase the system performance.

REFERENCES