A Bi-Objective Model and Efficient Heuristic for Hazardous Material Inventory Routing Problem

Nooraddin Dabiri and Mohammad J. Tarokh

Abstract—Two types of decisions that directly affect the performance of a distribution system is the vehicle routing and inventory assignment. These problems have been studied separately for many years. The inventory routing problem (IRP) is integration of above mentioned distribution problems. In the other words, IRP assumes each customer maintains a local inventory of a product and consumes a certain amount of that product each day. Each day a fleet of vehicles is dispatched over a set of routes to resupply a subset of the customers. Specifically, IRP determines inventory allocation and vehicle routing to objective to minimize the distribution cost. Therefore, the determination of hazardous materials distribution routes can be defined as a bi-objective inventory routing problem (BiIRP) while risk minimization accompanies the cost minimization in the objective function. In this paper we introduce an efficient bi-objective mixed integer linear programming model for IRP. For this purpose, we extend a well-known vehicle routing problem (VRP) formulation to consider the IRP assumptions. Also, we propose a binary chromosome representation to use in Multi-Objective GA algorithm. Finally, in order to evaluate the efficiency of proposed heuristic we design some novel experiments and solve them by both proposed algorithm and Cplex 12.2.

Keywords—Genetic algorithm, hazardous material distribution, inventory routing problem, vehicle routing.

I. INTRODUCTION

Supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandise is produced and distributed at the right quantities, to the right location, and at the right time, in order to minimize system-wide costs while satisfying service level requirements. This involves a set of management activities like purchasing, inventory control, production, sales and distribution. The overall goal of supply chain management is to integrate organizational units and coordinate flows of material, information and money so that the competitiveness of the supply chain is improved. For this purpose, implementing the reliable logistic system is vital. On the other hand, to remain competitive and profitable, products need to get to point of sale faster and more cost effectively than those of the competition. Inventory and transportation costs are two important cost components within enormous logistic systems. In order to deal with them, the vehicle routing problem (VRP) and the inventory management problem have been studied separately for many years. In order to reach more cost effective logistic system we should integrate VRP and the inventory related decisions.

An inventory routing problem (IRP) is a good example of integration between inventory and transportation problems which is cost effectively applicable in vendor managed inventory (VMI) environment. In the VMI model a vendor observes and controls the inventory levels of its customers, as opposed to conventional approaches where customers monitor their own inventory and decide the time and amount of products to reorder. In these types of distribution systems, the decision of how much inventory to maintain at the supplier and the retailers is affected by delivery times and amounts for the retailers, which in turn is affected by the capacity of the vehicles used for the deliveries. Thus, simultaneous decision making is important in these systems to obtain significant cost savings [1-2]. In the other words, The Inventory Routing Problem (IRP) combines routing and inventory as follows. Customers, based on their expected periodically demand, must be assigned to one or more days, and then a VRP problem must be solved for each day to assign vehicles to customers and determine routes for the vehicles, with a goal of minimizing the total delivery cost [3-5].

According to our finding, there are five publications which focus on literature survey of inventory routing problem. First effort was presented as book chapter by [6]. Second publication was a survey paper prepared by [7]. They called this problem as Dynamic Routing and Inventory Problems (DRIP). They define dynamicity as a situation where repeated decisions have to be made at different times within a planning horizon, and earlier decisions influence later decisions. This is a common feature in most of the IRP presented in the literature. Sarmiento and Nagi [8] provide third survey which was a review on integrated analysis of production–distribution systems. Recently another research has been done by [9]. They claim to focus on logistical overview of inventory routing problems, and very little researches have been mentioned about the application oriented works within the field. Finally, comprehensive review has been done in the

Nooraddin Dabiri is with K.N. Toosi University of Technology, Tehran, Iran. (Corresponding author: phone: +989124865385; fax: +982188674858; e-mail: n.dabiri@yahoo.com).

Mohammad Jafar Tarokh is with K.N. Toosi University of Technology, Tehran, Iran. (e-mail: mjtarokh@kntu.ac.ir).
published paper by [10]. Authors of last paper strongly emphasize on industrial aspect of inventory routing problem. We refer an interested reader to described five surveys for references to several of types and applications of IRP.

The hazardous materials transportation covers a large part of the economic activities of the industrialized countries. It has been estimated that four billion tons of hazardous materials are being transported annually at a worldwide level. Apart from its significant role in the economy of the industrialized countries, hazardous materials transportation raises concerns about the safety of the human and natural environment [11]. Therefore the consideration of the hazardous materials distribution as a bi-objective IRP may consequences many benefits.

In this paper we introduce an efficient bi-objective mixed integer linear programming (MILP) model for IRP in second section. In the third section we propose a binary matrix as chromosome representation to use in Multi-Objective GA (MOGA) algorithm as a solution approach. In order to evaluate the efficiency of proposed heuristic we design some experiments and solve them by both proposed algorithm and Cplex 12.2 in section four. Finally, we summarize the results of our research in the last section.

II. MODEL FORMULATION

A. Problem Statement

The IRP considered in this study is the problem of determining routes of vehicles and delivery quantities for each retailer over a given planning horizon which composed of several discrete time periods. The objective is to minimize the transportation and inventory holding costs simultaneously. Transportation costs consist of variable distance related cost and fixed cost which considered for each vehicle used in a period. Assumptions made in this study are given below:

1) There is a single warehouse supplying multiple retailers with products of a multiple type.
2) There is no limit on the availability of the products in the warehouse.
3) Vehicles are homogeneous, that is, vehicles have the same loading capacity.
4) A vehicle makes a single trip in a period and it may visit several retailers in each trip.
5) The set of retailers served by a vehicle may change throughout the planning horizon. Sequentially, retailers are to be partitioned into several groups in each period.
6) Transportation cost includes two components, traveling distance related cost and vehicle fixed cost.
7) The travel costs between the warehouse and each retailer and between all pairs of retailers are known, symmetric and constant over planning horizon.
8) The demand quantity of each retailer in each period is deterministic and known in advance, but may vary.
9) There is limited storage facility for inventory at the retailers.
10) Shortages are not allowed.

In order to formulate proposed IRP model we extend a basic VRP model that is firstly proposed by [12]. The notations and mathematical modeling for bi-objective inventory routing problem is given as follows. Unlike previously proposed formulations such as [13], in the following formulation we relax the vehicle indicator indices. Therefore our model is very useful when we use non-internal and non-much specified vehicles in the distribution fleet.

Notations for parameters and variables are illustrated in Table I and Table II, respectively.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N)</td>
<td>number of customers</td>
</tr>
<tr>
<td>(T)</td>
<td>number of periods</td>
</tr>
<tr>
<td>(i, j=0, 1, ..., N)</td>
<td>denote the indices of locations in which index 0 indicates supplier’s location</td>
</tr>
<tr>
<td>(t=1, 2, ..., T)</td>
<td>denotes the indices of the multiple period</td>
</tr>
<tr>
<td>(d_i)</td>
<td>demand of customer (i) at period (t)</td>
</tr>
<tr>
<td>(h_i)</td>
<td>holding cost per product unit at customer (i)</td>
</tr>
<tr>
<td>(f_i)</td>
<td>fixed usage cost per vehicle at period (t)</td>
</tr>
<tr>
<td>(c_v)</td>
<td>vehicle capacity</td>
</tr>
<tr>
<td>(r_{ij})</td>
<td>denotes transportation risk generated on the path from location (i) to location (j)</td>
</tr>
<tr>
<td>(C_i)</td>
<td>storage capacity of customer (i)</td>
</tr>
</tbody>
</table>

B. Mathematical Model

The following is a mixed integer programming formulation for the problem.

\[
\min TC = \sum_{t=1}^{T} \sum_{i=0}^{N} h_i \cdot I_{it} + \sum_{t=1}^{T} \sum_{i=0}^{N} \sum_{j=0}^{N} c_{ij} \cdot x_{ijt} + \sum_{t=1}^{T} f_i \cdot V_t
\]

\[
Z_{risk} = \sum_{t=1}^{T} \sum_{i=0}^{N} \sum_{j=0}^{N} r_{ij} \cdot X_{ijt}
\]

Subject to:

\[
I_{i,t-1} + D_i = I_i + d_i \quad \forall i, t
\]

\[
I_i \leq C_i \quad \forall i, t
\]

\[
\sum_{j=0, j \neq i}^{N} X_{ijt} \leq 1 \quad \forall i, t
\]
\[ \sum_{j=0, j \neq i}^{N} x_{ijt} = \sum_{j=0, j \neq i}^{N} x_{ijt} \quad \forall i, t \quad (6) \]
\[ \frac{1}{M} \cdot D_{ij} \leq \sum_{j=0, j \neq i}^{N} x_{ijt} \leq M \cdot D_{ij} \quad \forall i, t \quad (7) \]
\[ U_{ij} \cdot U_{ij} + q \cdot x_{ijt} \leq q - D_{ij} \quad \forall i, j | i \neq j \text{ and } t \quad (8) \]
\[ \sum_{j=0}^{N} x_{ijt} = V_{i} \quad \forall t \quad (9) \]
\[ D_{ij} \geq 0 \quad \forall i, t \quad (10) \]
\[ I_{ij} \leq 0 \quad \forall i, t \quad (11) \]
\[ x_{ijt} \in \{0, 1\} \quad \forall i, j \quad (12) \]

Equation (1) that is first objective function will minimize the distribution costs including inventory holding costs, variable transportation costs and vehicle usage fix costs. Hazardous material handling as second objective function is represented by Equation (2). Constraints (3) ensure inventory balance for each customer at the end of each period. For each period, constraints (4) bound the inventory level of the customers to their storage capacity. Constraints (5) make sure that a vehicle will visit a location at most once in a time period and constraints (6) ensure route continuity. If the delivery for a customer \( i \) at period \( t \) is equal to zero \((D_{ij}=0)\), then any vehicle should not visit it. This fact has been implemented by constraints (7). Constraints (8) serve two purposes. The first one is to ensure that the assigned amount for a vehicle is less than or equal to the vehicle’s capacity, and the second is to ensure the sub-tour elimination for each vehicle in each period. Constraints (9) will determine number of vehicles is used in each period. Constraints (10)-(12) are the domain constraints.

III. Solution Approach

Inventory routing problem is NP-hard [1]. As a consequence exact solution methods cannot find the optimum solution even for moderate size problems. Alternatively several heuristic approaches have been developed within the last two decades aiming at identifying near optimal solutions. Along with the most extensively adopted techniques are: simulated annealing, ant colonies, genetic algorithm, neural network, particle swarm optimization and etc. Here we implement the genetic algorithm (GA) to solve described problem in the previous section. GA is a stochastic search technique that explores the problem domain by maintaining a population of individuals, which represents a set of potential solutions in the search space. GA attempts to combine the good features found in each individual using a structured yet randomized information exchange in order to construct individuals who are better suited to their environment than the individuals that they were created from. Through the evolution of better and better individuals, it is anticipated that the desired solution will be found.

In this paper we just introduce a proper binary chromosome representation and other aspects of MOGA method will be same as others in literature. More specifically proposed chromosome is based on deciding when a customer will be visited among planning periods. In the other words, proposed chromosome is a binary matrix that indicates in each period which customers should be visited by vehicles.

Figure 1 illustrates an example of the chromosome representation for a problem having 5 customers and 5 periods. For instance customer 3 will be visited in the first and last periods. According to the visiting chromosome we could simply determine delivery matrix (inventory assignment). The amount to be delivered depends on whether there will be visit in the subsequent period or not. The total delivery to customer \( i \) in period \( t \) is the sum of all the demands in period \( t, t+1, ..., k-1 \) where the next visit will be made at period \( k \). After delivery scheduling determination we should decide about vehicle routing, for this purpose in this paper we use well-known saving algorithm proposed by [14] in each period. Since the saving algorithm does not require fleet size as input parameter, so it’s completely compatible with proposed decision problem in previous section. Here we ignore the description of the MOGA, because it’s very famous so for more illustration we refer researchers to [15].
IV. COMPUTATIONAL ANALYSIS

In this section, we attempt to evaluate proposed multi-objective GA algorithm which discussed in previous section. According to our design of experiments, customers are allocated in a square of 50×50 distance units and their coordinates are generated using a uniform distribution within these limits. The depot is located in the middle of the square. The Euclidean distance between each pair of locations was computed to obtain a complete graph whose path weights are the travel distance. Traveling costs \( (c_{ij}) \) was calculated by rounding up corresponding travel distance and multiplying it by transportation cost rate. Associated risk for each path \( (r_{ij}) \) is generated using a uniform distribution within \((0, 10)\). In this paper we suppose transportation cost rates are constant and equal to 2. Holding costs are different for each customer and have absolute normal distribution with a mean of 0.03 and a standard deviation of 0.05. A constant cost value of 10 for fixed the vehicle usage cost in each period \( (f_t) \) is assumed. For each combination of periods and products, customer demands \( (d_{it}) \) are generated using a uniform distribution from 5 to 25. It is assumed that vehicle fleet is homogenous and each vehicle capacity is 100 items. Also, each customer has a storage capacity of 150 items. We generate three levels of \( N \) (10), (20), and (30) and two levels of \( T \) (5) and (7). For each problem setting defined by a combination of \( N \) and \( T \), we randomly generate two replicates.

The naming convention used for the test problems starts with two digits that are assigned for the number of customers. After a hyphen, a digit refers to presenting the number of the planning periods. Finally, the replicate number is given at the last digit after a hyphen. Thus, the problem 10-5-1 represents the first replicate of a test with 10 customers and planning periods of 5. We have implemented the run numerical examples on a personal computer with 2.53 GHz Core-2-Due CPU and 3 GB RAM.

As stated earlier, Multi-objective GA toolbox of the Matlab software is used to solve the proposed model. For this purpose after many examinations we apply following setting to toolbox required arguments. We set population size 150, and uniform population creation. Also, tournament operator is used as selection function. Two point with rate 0.8 and uniform operators with rate 0.05 are used as crossover and mutation functions, respectively. Well-known distance crowding function with fraction rate 0.35 is used for controlling the diversity of Pareto solutions. Also, we set 1200 seconds as stopping criteria.

In the literature many algorithms have been developed to find the exact solution for Pareto set. The \( \varepsilon \)-constraint method was proposed by [16] in 1971, this method is one of the best known approaches for solving multi-objective problems. In this method one objective is selected as the main objective and other objective are transformed into constraints. The drawback of the conventional \( \varepsilon \)-constraint method lies on efficiency of its Pareto frontiers and there is no guarantee for the efficient solutions and it may generate inefficient solutions. The augmented \( \varepsilon \)-constraint (AUGMECON) method is novel version of the conventional \( \varepsilon \)-constraint method that provides efficient Pareto frontiers. We refer an interested reader to find the description of AUGMECON to see [17]. In this paper, we evaluate the efficiency of the proposed algorithm comparing its results with pay-off table of efficient Pareto solution obtained by AUGMECON method.

Table III shows detailed computational results of the experiments. At first, the AUGMECON method for proposed model was implemented in GAMS software using Cplex 12.2 to obtain efficient two Pareto solution. Then middle point is calculated by average of above points. Also, MOGA heuristic was applied to 12 test problems and two of the obtained Pareto solutions frontiers were selected based on being best of two objective functions. The selected solutions are used to calculate the improvement percent according Eq. (13).

\[
\text{improvement \%} = 100 \times \left( \frac{OF_{\text{AUGMECON}} - OF_{\text{MOGA}}}{OF_{\text{AUGMECON}}} \right)
\] (13)

According to the results, it can be said that if we use MOGA algorithm the first objective function will improve about 25% in average against the use of AUGMECON method. But, second objective function will be worse about 7% in the result of the MOGA rather than AUGMECON method. This disadvantage in is not considerable so much, because according to Figure (2) improvement percentage will increase by raising the number of binary variables. Therefore we could say the results demonstrate the practicability of the proposed bi-objective model as well as the proposed solution algorithm.
The Inventory Routing Problem (IRP) combines routing and inventory as follows. Customers, based on their expected periodic demand, must be assigned to one or more days, and then a VRP problem must be solved for each day to assign vehicles to customers and determine routes for the vehicles, with a goal of minimizing the total distribution cost. In this paper we propose an extension of IRP models in order to consider transportation risk as well as its cost. For this purpose, we extend a well-known VRP formulation to satisfy IRP assumptions. Also, we implement MOGA solution approach to solve proposed bi-objective mathematical model. Computational results illustrate the practicability of the proposed bi-objective model as well as the proposed solution algorithm.

REFERENCES


