Abstract—This paper presents the system identification of a half-scale platform of MLRS using Lagrange’s equation and SolidWorks’ simulation analysis tool. The system parameters were identified and the model was developed using Lagrange’s equation. Then the model was employed to simulate the multi-sine input response and the simulation results were compared to the experimental results. It was found that the model was developed using Lagrange’s equation having percentage best fit of 94.81% for the azimuth angle control and 92.39% for the elevation angle control. Overall, this method could be accepted with a good percentage best fit that compared the experimental results.

Keywords—System identification, Lagrange’s equation, MLRS, multi-sine signals

I. INTRODUCTION

This paper was research and development a half-scale platform of Multi-Launcher Rocket System (MLRS) at laboratory in Defence Technology Institute of Thailand. In order to design the controller for any dynamical system, a suitable dynamical model of the system needs to be formulated and its parameters need to be accurately identified [1-4]. Required model parameters were obtained with various methods. Some parameters were obtained from handbooks and manufacturer specifications, and some parameters needed to be obtained through experiments. However, other parameters needed to be obtained through computer simulation. This also holds for a half-scale platform of MLRS, this paper proposes the system identification using Lagrange’s equation and SolidWorks’ simulation analysis tool were identified and the model was developed, then the model was employed to simulate the multi-sine input signal and the simulation results were compared to the experimental results for validation the model. This paper was organized in the following manner: Section II Plant; Section III System identification; Section IV Root locus and stability analysis; Section V Validation; Section VI Experimental results and discussion; finally, Section VII is the conclusion.

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II. PLANT

The plant used of this research is the platform of a half-scale of MLRS as shown in Fig. 1. The platform consists of seven major parts as following: (i) The base use for supporting that has the dimension 1040 mm., 1120 mm. and 260 mm. in width, length and thickness respectively and the total weight is 76.188 kg; (ii) The cradle use for construction of all azimuth angle parts that has dimension 571 mm., 882 mm. and 76 mm. in width, length and thickness respectively and the total weight is 143.623 kg; (iii) The elevation platform is the all of the elevation control devices and rocket pod that has dimension 985 mm., 1380 mm. and 234 mm. in width, length and thickness respectively and the total weight is 93.811 kg; (iv) The cylinder is a bidirectional cylinder type that has the piston diameter is 63 mm., piston rod diameter 36 mm., stroke 200 mm., and piston area ratio 1.75:1; (v) The pressurized fluid flow is control by electronic control valve. This control valve is the proportional and directional type that have input voltage ±10 V dc with current 4-20 mA.; (vi) The pressurized pump and hydraulic motor are regulate fluid source of up to 2300 psi or 160 bars;

Fig. 1 Plant of a half-scale platform of MLRS

(vii) The controller using the programmable logic controller (PLC). Also this research studies together with SolidWorks’ simulation to simulate and determine the parameters use for Lagrange’s equation as shown in Fig. 2 and Fig. 3.
III. SYSTEM IDENTIFICATION

A. Lagrange’s Equation for the Dynamic Model

It is well known that the model of the dynamics of an n-joint rigid robot manipulator [1-4] can be written in the joint space using the Lagrange’s equation follow as “(1),” then this research implement this method for the model of the dynamics of a half-scale platform of MLRS can be written in the motion equation as

\[ \tau = B(q)\dot{q} + C(q, \dot{q})q + g(q) + F_v \text{Sin}(q) + F_s \dot{q} \]  

where \( q \in \mathbb{R}^n \) is the vector of relative joints displacements between the links, \( \tau \in \mathbb{R}^n \) is the torque vector, \( B(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is the centripetal and coriolis matrix, \( F_v \in \mathbb{R}^{n \times n} \) is the viscous friction matrix, \( F_s \in \mathbb{R}^{n \times n} \) is the static friction matrix, and \( g(q) \in \mathbb{R}^n \) is the vector of gravitational torques.

B. Lagrange’s Equation for the Azimuth Angle Control

The model of the azimuth angle control of a half-scale platform of MLRS can be written by Lagrange’s equation and determine the parameters of the plant using SolidWorks’ simulation as shown in Fig. 2.

From the results of SolidWorks’s simulation show that the CG at \( X=101.94 \text{ mm}, \ Y=156.48 \text{ mm}, \ Z=54.52 \text{ mm}, \) the length of CG to joint \( (l) = 99 \text{ mm}, \) the mass \( (M) = 143.62 \text{ kg}, \) the gravitational \( (g) = 9.81 \text{ m/s}^2, \) and the inertia \( (J) = 26.85 \text{ kg.m}^2, \) and gear ratio=80:10, then situation as following.

\[ J \ddot{\phi} + Mgl \sin \phi = \tau_1 \]  

\[ 26.85(\ddot{\phi} + (143.62 \times 9.81 \times 0.99) \sin \phi) = (80/10)\tau_1 \]  

\[ 3.356\ddot{\phi} + 174.35\sin \phi = \tau_1 \]  

Equation (4) is a non-linear equation. This research was considered in the range of operating and linearized [10] by \( \sin \phi \approx \phi \). Thus,

\[ 3.356\ddot{\phi} + 174.35\phi = \tau_1 \]  

Then, take Laplace transform as below

\[ 3.356s^2\phi(s) + 174.35\phi(s) = \tau_1(s) \]  

Thus, determine the transfer function that including controller and the output has a unit in degree as below.

\[ G_{\phi}(s) = \frac{\phi(s)}{V_m(s)} = \frac{18.45}{s^2 + 51.9517} \]

C. Lagrange’s Equation for the Elevation Angle Control

The model of the elevation control system of the half-scale of MLRS can be written as Lagrange’s equation and determined the parameters by SolidWorks’ simulation as shown in Fig. 3.

From the Simulation result of the Elevation Control System and found that the CG at \( X=-59.18 \text{ mm}, \ Y=83.91 \text{ mm}, \ Z=-109.22 \text{ mm}, \) the length of CG to joint \( (l) = 290 \text{ mm}, \) the mass \( (M) = 93.812 \text{ kg}, \) the gravitational \( (g) = 9.81 \text{ m/s}^2, \) and the inertia \( (J) = 17.62 \text{ kg.m}^2. \) Thus,

\[ J \ddot{\phi} + Mgl \sin \phi = \tau_2 \]  

\[ 17.62(\ddot{\phi} + (93.812 \times 9.81 \times 0.290) \sin \phi) = \tau_2 \]  

\[ 289.266\ddot{\phi} + 62.17\sin \phi = \tau_2 \]  

Equation (10) is a non-linear equation. This research was considered in the range of operating and linearized [10] by \( \sin \phi \approx \phi \). Thus,

\[ 17.62\ddot{\phi} + 62.17\phi = \tau_2 \]  

Then, take Laplace transform as below

\[ 17.62s^2\phi(s) + 62.17\phi(s) = \tau_2(s) \]  

Thus, determine the transfer function that including controller and the output has a unit in degree as below.

\[ G_{\phi}(s) = \frac{\phi(s)}{V_m(s)} = \frac{31.55}{s^2 + 15.174} \]
D. State Space Model of a Half-scale Platform of MLRS

From the transfer function in “(7)” and “(13)” that using the Lagrange’s equation of the azimuth angle control and the elevation angle control respectively. Then determine the state space equation [6-10]. Now let,

\[
\begin{align*}
x_1(t) &= \theta(t), \\
x_2(t) &= \dot{\theta}(t), \\
x_3(t) &= \phi(t), \\
x_4(t) &= \dot{\phi}(t)
\end{align*}
\]

Then select \( x_1(t), x_2(t), x_3(t), \) and \( x_4(t) \) as state variables. And complete the form of state equations shown as “(14)” and “(15).” Thus, the MIMO state space variable as becomes

\[
\begin{align*}
x(t) &= Ax(t) + Bu(t) \quad (14) \\
y(t) &= Cx(t) + Du(t) \quad (15)
\end{align*}
\]

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-51.9517 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -15.174 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} +
\begin{bmatrix}
18.45 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 31.55
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
\]

IV. Root Locus and Stability Analysis

From the state space model can analyze the root locus of the plant found that the azimuth angle control systems has poles at \( \sigma = 7.2083i \) on the imaginary axis and the elevation angle control system has poles at \( \sigma = 3.8954i \) on the imaginary axis as shown in Fig. 4. Thus the system of a half-scale of MLRS is an unstable open loop plant.

The state space model can analyze stability using a unit step input to determine the step response of the open loop system, the result shown as Fig. 5 that consists of output1: Azimuth angle (From input1) and output2: Elevation angle (From input2) and the results of step response of open loop system is an unstable system. Therefore, the system of a half-scale of MLRS is an unstable open loop plant.

V. Validation

The multi-sine signal given in “(16)” and “(17)” as shown in Fig. 6 was used as an input signal to the plant for model validation and experiment [4-5]. The signal was generated using three different frequencies as following.

\[
u(k) = \sum_{i=1}^{p} a_i \cos \omega_i t_k \quad (16)
\]

\[
V_{in}(k) = \cos 0.5 t_k + \cos 2 t_k + \cos 5 t_k \quad (17)
\]

VI. Experimental Results and Discussion

The comparison of the experimental results and simulation results of the model using Lagrange’s equation that excited by multi-sine input signals are given in Fig. 6, and the results shown as Fig. 7. As the time response, the experimental results are slow and show small differences. This is due to the abilities of electronic valve to open and close at its capabilities rate.

The system identification using Lagrange's equation of
azimuth angle control having percentage best fit of 94.81% and the system identification of elevation angle control having percentage best fit of 92.39% respectively.

VII. CONCLUSION

The system identification of a half-scale platform of MLRS using Lagrange’s equation and SolidWorks’ simulation were identified, then the model was employed to simulate the multi-sine input response and the simulation results were compared to the experimental results. It was found that the model can be accepted and meet to the requirements.

The simulation and experimental results are given small differences. This is due to the abilities of electronic valve to open and close at its capabilities rate. Further investigations have to be conducted to explain and improve the accuracy of the model.

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